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COMPUTER PROGRAMS FOR PREDICTION OF LIGHTNING INDUCED VOLTAGES IN AIRCRAFT ELECTRICAL CIRCUITS

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General Electric Corporate Research and Development

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COMPUTER PROGRAMS FOR PREDICTION OF LIGHTNING INDUCED VOLTAGES IN AIRCRAFT ELECTRICAL CIRCUITS

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This report describes a computerized program to define the induced circuit voltage within an aircraft electrical system due to a lightning strike on the aircraft. One routine of the program (DIFFUSION) calculates the effect of magnetic fields caused by current on the aircraft skin. The other routine (APERTURE) calculates the magnetic field that enters the aircraft because of an opening. The induced voltages are then calculated for any given electrical circuit.

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20. Abstract (cont'd)

The program has defined geometrical configurations for a fuselage, rectangular wing, and empennage sections. A subroutine calculates the current distribution on the skin of the section being analyzed. The program input current and output voltage are in the time domain.

FOREWORD

The work reported in this document was conducted by the Physics and Electronics Engineering Laboratory in Corporate Research and Development of the General Electric Company in Schenectady, New York, on "Computer Programs for Prediction of Lightning Induced Voltages in Aircraft Electrical Circuits," sponsored by the Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, under Contract F33615-74-C-3068, J.O. Number 19870275 from 1 February 1974 to 30 November 1974. Mr. Dong G. Kim (FGL) was the AFFDL Project Engineer. This document was submitted to AFFDL in February 1975.

This report describes a computerized program to define the induced circuit voltage within an aircraft electrical system due to a lightning strike on the aircraft. One routine of the program (DIFFUSION) calculates the effect of magnetic fields caused by current on the aircraft skin. The other routine (APERTURE) calculates the magnetic field that enters the aircraft because of an opening. The induced voltages are then calculated for any given electrical circuit. The program has defined geometrical configurations for a fuselage, rectangular wing, and empennage sections. A subroutine calculates the current distribution on the skin of the section being analyzed. The program input current and output voltage are in the time domain.

Contributions to this contract effort from Mr. J.E. Houtz of AFFDL/FGL is gratefully acknowledged.

Information on this document and on how to obtain a card deck listing of the program may be obtained from Mr. Kim, AFFDL/FGL, Wright-Patterson Air Force Base, Ohio 45433.

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Section 1

BACKGROUND AND OBJECTIVES

LIGHTNING INDUCED VOLTAGE IN AIRCRAFT ELECTRICAL CIRCUITS

One of the trends in the design of modern aircarft is toward the use of miniaturized solid-state electronics in avionics, automatic flight control, and other functions. The decrease in weight and power consumption which these devices afford has enabled improvements in performance and economy, so the trend is likely to become widespread in the design of new military and civilian aircraft. Because of the inherently small size and low operating power levels required by miniaturized solid-state electronics, however, these components have been found to be more vulnerable to electromagnetic interference than their vacuum tube counterparts of an earlier day. Such interference may result from on-board systems such as radio transmitters or relay operation, or from external sources such as lightning or nuclear electromagnetic pulse (NEMP).

Incidents have been reported, for example, in which solid-state electronics have been upset or permanently damaged as a result of lightning strikes to aircraft (Refs. 1 and 2). A number of research programs have been conducted by various Government agencies or research laboratories to determine the extent of lightning related interference and the mechanisms by which it occurs in aircraft electrical circuits (Refs. 3-6), and it has been learned that electromagnetic fields caused by lightning may appear inside typical aircraft and induce transient surge voltage in electrical wires and cables. These lightning-induced voltages are in addition to those which may enter aircraft electrical circuits as a result of direct lightning stroke contact with externally mounted electrical components such as navigation lights or antennas.

Malfunction of sensitive electronics may occur if the induced voltage exceeds its overvoltage withstanding capability or if the accompanying induced current surge results in the dissipation of excessive power in semiconductor junctions. Since lightning electromagnetic fields usually permeate the entire aircraft, redundant systems are also susceptible and may not provide their intended backup capability.

Heretofore, most lightning protection design has been for control of the direct effects of lightning, such as fuel ignition and structural strength degradation, or directly conducted surge voltages and currents arising from strokes to navigation lights or antennas. However, the increasing dependence of critical navigation and flight control functions on solid-state microelectronics has resulted in recognition of the need for protection against the indirect effects of lightning.

Test techniques and equipment have been designed for subjecting complete aircraft to simulated lightning strokes so that the degree of susceptibility of various individual circuits/systems can be determined. Such techniques allow measurement of actual induced voltages in these circuits for comparison with known component withstand levels, to determine the need for additional protective measures. To avoid expensive retrofit programs, however, lightning protection should be designed into each aircraft system from the start. This means that designers must have information concerning the expected susceptibility of particular circuits while they are still on the drawing board, when there is as yet no aircraft on which to run tests.

To fulfill this need, a program was initiated by the Air Force Flight Dynamics Laboratory at Wright-Patterson Air Force Base to develop a computerized analysis technique for calculating voltages expected to be induced in typical aircraft circuits by lightning stroke currents flowing through the aircraft. The overall objective was to develop a computer program, readily and economically usable by aircraft designers, to assess the impact of various structural and electrical system design configurations on lightning susceptibility, and thereby to provide a tool by which design optimization and tradeoff studies can be made from the standpoint of lightning protection.

The analytical approach followed was based upon a preliminary attempt, made under an earlier National Aeronautics and Space Administration contract (Ref. 5), which showed promise when compared with actual experimental measurements. After some introductory discussions of lightning induced voltage mechanisms, this report describes the analytical steps applied and the computer programs developed to calculate voltages induced in electrical conductors at various locations inside a complete aircraft.

DIRECT COUPLED VOLTAGES

Directly coupled voltages occur as a result of direct contact of lightning strike currents with exposed (external) electrical assemblies, such as antennas and navigation lights. If a lightning flash punctures a lamp globe or antenna housing so that direct contact may be made with a filament or antenna element, a portion of the lightning current may be conducted into associated electrical wiring. This voltage will be accompanied by a voltage surge limited in amplitude by the insulation breakdown voltage level of the electrical assembly or associated wiring, whichever is less. Unless external protection is applied to prevent puncture of the external assembly, it is often damaged beyond operational capability. Even if this damage is acceptable, however, the surge voltages and currents which proceed into associated wiring are usually hazardous to connected equipment such as power control or communication electronics.

Thus, protection against these surges must be provided. The magnitude of these conducted surges and the adequacy of protective devices designed to

control them must usually be evaluated by full-scale simulated lightning tests of the external assemblies in question. Government specifications for some of these devices or protective equipment are now in existence (Ref. 7). Directly coupled voltage and current surges are considered direct effects were not dealt with further in this program.

INDIRECT COUPLED VOLTAGES

The other mechanism by which lightning can affect aircraft electrical and avionics systems is by the generation of magnetically induced and resistive voltagerising within aircraft electrical circuitry. To describe the manner in which induced voltages occur it is first necessary to consider the mechanisms by which magnetic fields appear inside an aircraft.

For a long conductor carrying a current, i, and whose return path is far removed, the average field intensity at a distance, r, from the conductor is

$$H = \frac{i}{2\pi r} \qquad (1)$$

as shown in Figure 1.

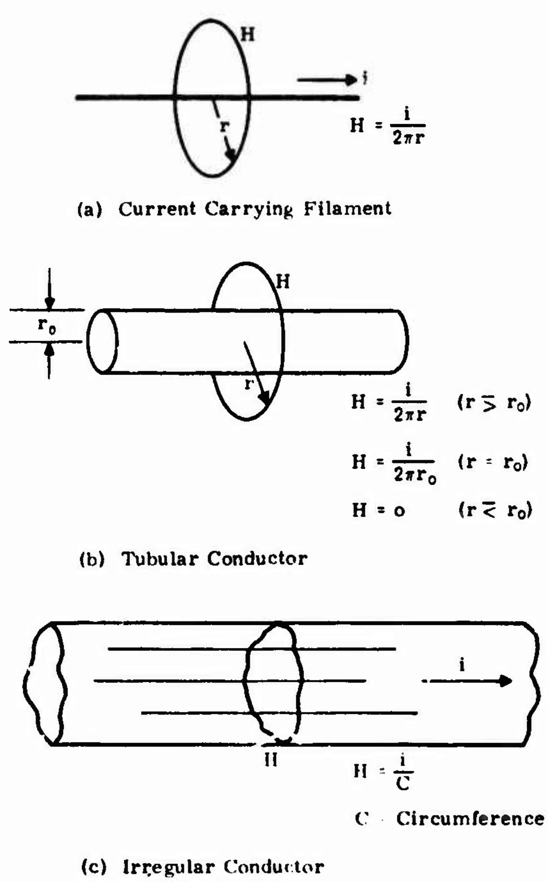

Figure 1. Magnetic Fields Around Current Carrying Conductors

If instead of a solid wire the current were carried on a hollow tube of radius r_o, as shown on Figure 1(b), the field intensity at radius $r < r_o$, would be

$$H = \frac{i}{2\pi r} \tag{2}$$

and at the surface of the tube where r equals r_o the field intensity would be

$$H = \frac{i}{2\pi r_0} \tag{3}$$

Since the circumference of a tube is

$$C = 2\pi r_0 \tag{4}$$

it follows that the field intensity at the surface of a tube is

$$H = \frac{i}{C} \tag{5}$$

The average current density at the surface of the tube is also equal to the total current divided by the circumference:

$$J_{AVE} = \frac{i}{C} \tag{6}$$

If the conductor is not cylindrical, as shown in Figure 1(c), the field intensity at different points on the surface will be different. Field intensity will still be equal to the total current divided by the circumference:

$$H_{AVE} = \frac{i}{C} \tag{7}$$

The actual field intensity will be greater than average at points where the radius of curvature is less than average, and less than average at points where the radius of curvature is greater than average, as shown in Figure 2.

For example, in a wing carrying lightning current, the leading and trailing edges have radii of curvature much smaller than average. Field intensity along the leading and trailing edges should then be quite high compared to the field intensity along the top or bottom surfaces.

Since both the average current density, J_{AVE}, and the average field intensity, H_{AVE}, are equal to the total current divided by the circumference,

$$J_{AVE} = H_{AVE} = \frac{i}{C} \tag{8}$$

it follows that the tangential field intensity at the surface of a conducting object is equal to the current density at that point. This is in fact true, at least for transient currents. The relation is not true for d-c currents or transients sufficiently slow that appreciable magnetic fields penetrate the skin.

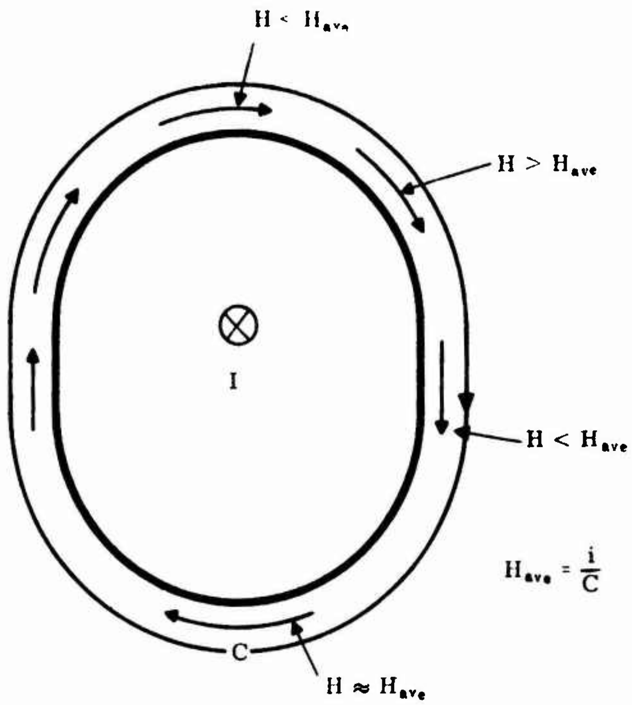

Figure 2. Field Intensity Versus Radius of Curvature

The orientation of the H field vector is always at right angles to the direction of the current vector, as shown in Figure 1.

While small gaps in the structure direct the current around the gap, the magnetic field is virtually unaffected, except directly on the surface and on a length scale that is small compared to dimensions of the gap interrupting the current flow (see Figure 3).

So far, only the field external to the aircraft has been dealt with. Even if the aircraft has an electrically continuous metallic skin, some magnetic flux can appear within the aircraft as lightning current diffuses through the skin to the inside surface. Cancellation effects will eliminate this flux in perfectly symmetrical cases, such as a cylinder with uniform skin current distribution, but in other cases some net diffusion flux may exist inside. The interior field is generally characterized as having a slower time to crest than the exterior field as well as a lower amplitude; this is illustrated in Figure 4.

If apertures exist in the aircraft skin, a portion of the external magnetic flux surrounding all of the current flowing through the aircraft will leak inside through these apertures, as shown in Figure 5. This is known as aperture flux; and it appears inside much sooner than the diffusion flux, since its velocity is unimpeded, and has a higher rate of change, similar to that of the total lightning current. Aperture flux is usually more localized than diffusion flux, and in the vicinity of apertures it may have a much higher amplitude than the diffusion flux.

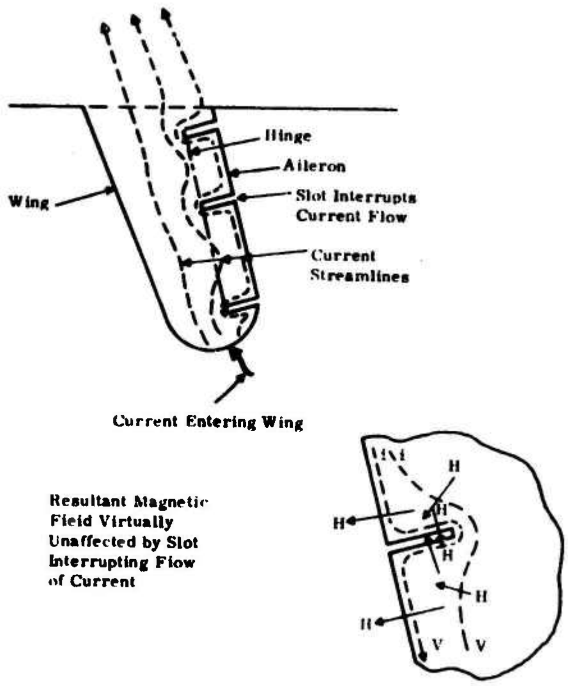

Figure 3. Current Flow and Magnetic Field Around Structural Gaps

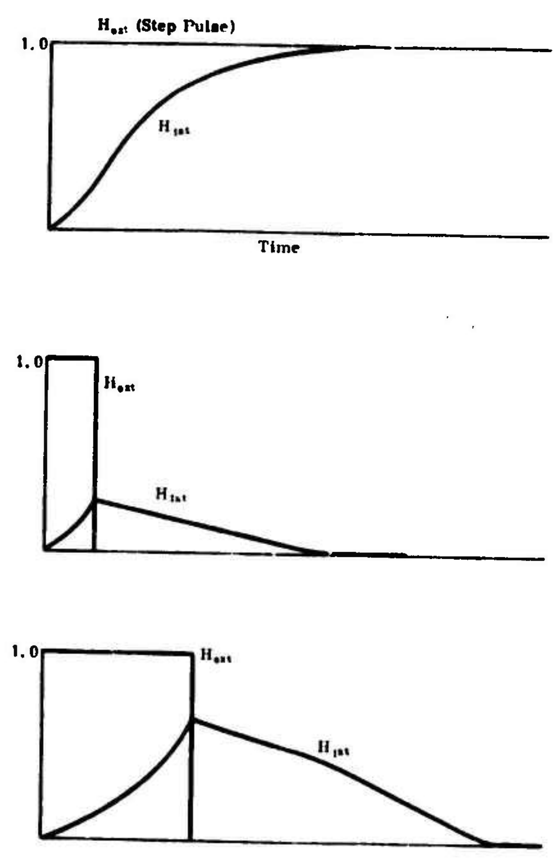

Figure 4. Internal Diffusion Fields as a Function of External Fields

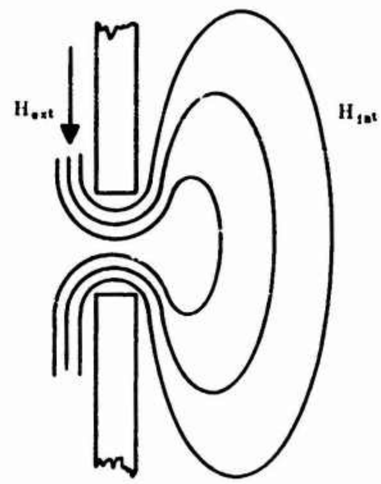

Figure 5. Aperture Fields

An aperture can be described in terms of an equivalent magnetic dipole (Figure 6). In Figure 6(a), current streamlines are seen being diverted around a hole in a current carrying sheet. A filamentary dipole producing the same magnetic effects as the diverted current flow would be as shown on Figure 6(b). The magnetic field pattern produced by such a dipole is the same as the classic magnetic field produced in the near field zone by a magnetic dipole, and is shown in Figure 7. The farther one is from the opening, the less is the field intensity, decreasing approximately as the third power of the distance, for distances that are large compared to the size of the opening.

The changing internal magnetic fields link electrical wires and cables inside the aircraft, inducing voltages therein. The induced voltages are related to the lightning current by inductive transfer functions (Ref. 3) in accordance with Faraday's law. Since the airframe is composed of inactive circuit constants, the transfer function for diffusion flux coupling should be a constant inductance for any lightning waveform, relating the portion of lightning current appearing at the inside surface of the skin to the voltage it induces in a circuit. The transfer function relating voltages induced into a circuit by the aperture flux is probably more complex because of less uniform field patterns and aperture geometries.

In addition to the magnetically induced voltages, the resistance of the metallic skin will permit resistive voltage differences in the skin (or structure) along the path of lightning current flow. If an aircraft electrical circuit employs the structure as its return path, then this resistive voltage enters the circuit, in series with the magnetically induced voltage in the same circuit and any other (normal) steady-state operating voltages present. Capacitively coupled voltages may also be produced in these circuits; however, the essentially uniform conducting skin of metallic aircraft keeps potential differences among structural elements low, thereby limiting the voltages which can be electrostatically coupled to interior electrical circuits. In

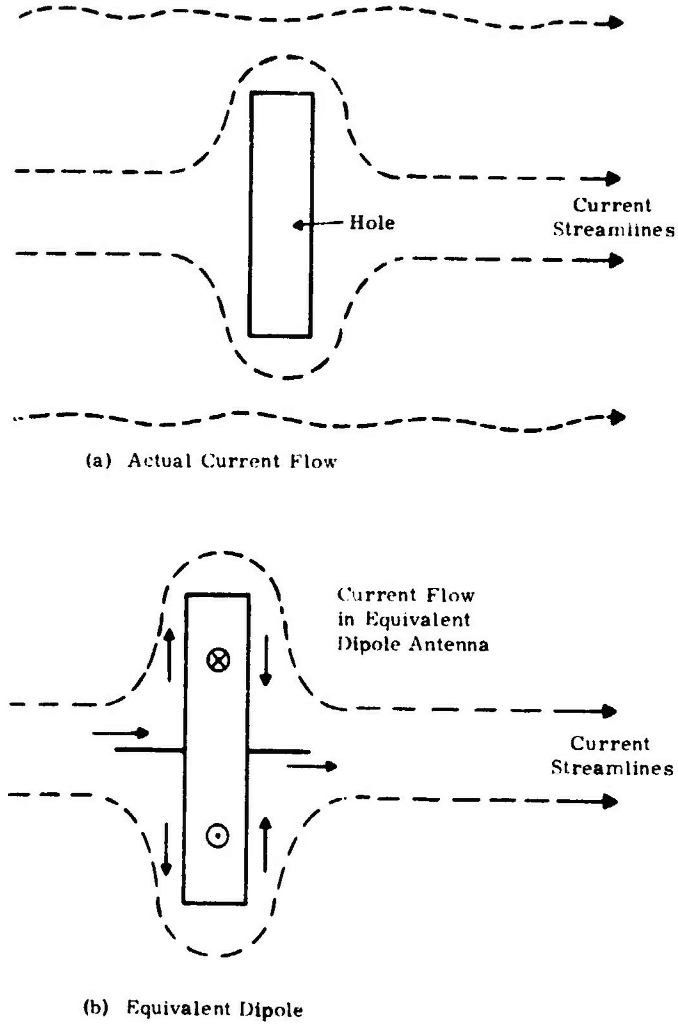

Figure 6. Development of the Equivalent Magnetic Dipole

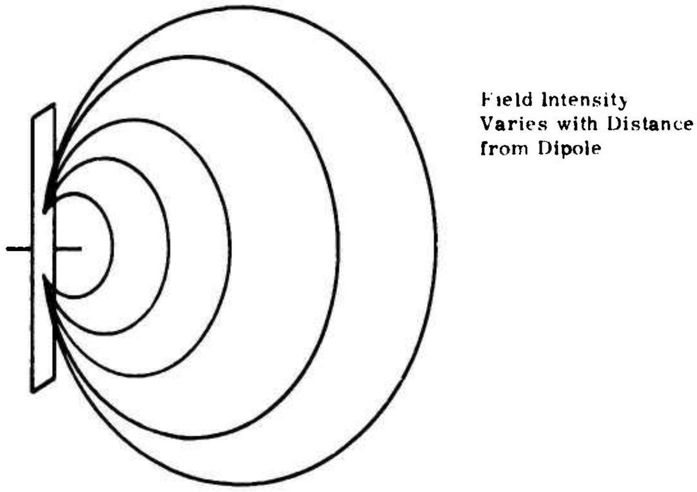

Figure 7. Field Pattern Due to an Aperture Dipole

practice, experimental measurements have shown magnetic and resistive components to be the most predominant (Refs. 3-6).

The combination of the resistive and magnetic components of induced voltages should therefore be expressible as follows:

$$e_{oc} = R_s i_L(t) + M_1 \frac{d[(1-e^{-\alpha t})\, i_L(t)]}{dt} + M_2 \frac{di_L}{dt} \qquad (9)$$

where:

e_{OC} = voltage induced in the circuit (using the airframe as return)

R_S = effective structural resistance

M_1 = diffusion transfer inductance between lightning current flowing on the inside surface of the skin and the particular electrical circuit

M_2 = aperture transfer inductance between the total lightning current flowing through the aircraft and the particular aircraft circuit

$i_L(t)$ = lightning current (a time varying function)

α = reciprocal of the time constant of current penetration into the aircraft skin

Of course, circuit transmission line and termination impedance characteristics as well as secondary induced effects may change the induced voltage appearing at a particular point from that predicted by Equation 9. Equation 9 is therefore most appropriately viewed as representing the induced source voltage driving the distributed aircraft circuit.

In the first experimental programs (Refs. 3-5), transfer functions derived from induced voltage data indicated that most of the enclosed magnetic flux was of diffusion origin, and the M_2 term of Equation 9 was not necessary for this equation to adequately describe the measured induced voltage waveforms. The work of Refs. 3, 4, and 5 was conducted on an F89J fighter aircraft, however, which has few apertures. Subsequent work on different aircraft (Ref. 6) with more apertures showed evidence of much greater aperture field coupling into aircraft circuits; this mode was often more predominant than either the diffusion magnetic or resistive mechanisms. At the conclusion of the F89J tests, work was initiated on a completely analytical technique to arrive at the same transfer functions (Ref. 5). This involved a mathematical representation of an F89J wing and an electrical circuit conductor inside. Some simplifying assumptions relating to wing geometry and lightning current flow were made in this attempt, and the magnetic flux linking the conductor and its airframe was calculated as a function of an assumed

lightning current filament in the wing skin. The contributions from a large number of such filaments, assumed to comprise the wing, were summed to obtain the total magnetic flux linking the conductor and its airframe return. From this, the transfer inductance, M_1, was derived. The resistive transfer function, R_s, was calculated as a function of geometry and material resistivity. The resulting values of R_s and M_1 compared well with corresponding values derived from measured induced voltages on a circuit inside the F89J wing. The work accomplished in this program, particularly that dealing with diffusion coupled voltages, is based on this preliminary approach.

PROGRAM OBJECTIVES

The basic objective of this program was to develop computerized analytical models to determine possible lightning induced voltages in aircraft electrical circuits. Specific requirements for these models were that they represent the major airframe sections of a complete aircraft, including fuselage, wing, horizontal stabilizer, and vertical stabilizer.

Another goal was to incorporate as many refinements over the original model of Reference 5 as possible. The desired improvements included:

1. Calculation of the actual lightning current distribution throughout the circumference of each major section. (The original model assumed a uniformly distributed current.)

2. Representation of circuit conductors of different lengths than that of the major airframe section itself.

3. Location of the circuit conductor anywhere inside the airframe, instead of along its axis of symmetry only.

4. Calculation of voltages induced by aperture flux, such as would penetrate holes, windows, and access doors. (The original model assumed a completely enclosed airframe.)

5. Calculation of the effect of varying one circuit location or airframe geometry parameter while holding the other constant.

To the extent possible within the program resources, it was also desired to represent internal structural elements, such as spars, ribs, and bulkheads, and to accommodate more complex electric circuit configurations such as shielded cables, wire-to-wire (independent return), and individual circuit impedance characteristics.

Upon completion of each basic model, its mathematical equations and validity were to be verified by having them represent simple geometries for which textbook solutions are available and aircraft geometries for which test data are available.

The computerized models were to be programmed in FORTRAN extended version IV for execution with punched cards on the U. S. Air Force CDC 6600 computer at Wright-Patterson Air Force Base, Ohio. The program was to be delivered as a punched card deck. A user's manual and a final technical report were also to be delivered.

BASIC APPROACHES

As previously discussed, lightning induced voltages in aircraft electrical circuits occur because time varying aperture and diffusion magnetic fluxes exist inside the airframe. Aperture flux penetrates through openings such as windows and access doors in the aircraft structure. Diffusion flux appears inside as lightning currents diffuse through the thickness of the metallic skin and appear on its inside surface.

Because the methods by which these fluxes enter the airframe are fundamentally different, it was decided that completely separate models should be developed to represent the diffusion and aperture coupling mechanisms. The diffusion model is based on the original approach of Reference 5, which assumed no apertures, whereas the aperture model is based on treatment of a single aperture which opens into a relatively small, confined space in the airframe. Contributions to aperture flux from individual apertures are considered of greatest interest, because the flux entering from one aperture is frequently segregated from that entering through other apertures by the presence of spars, ribs, bulkheads, and other interior walls, which act as electromagnetic shields.

For most airframe or circuit situations it is not intuitively obvious which of these two fluxes induces the greater voltages. Therefore, it will be necessary for designers to utilize both models for a complete evaluation of possible induced voltages; but as experience is gained, situations will become apparent which heavily favor one or the other model.

Section 2

DIFFUSION MODEL

INDUCED VOLTAGE THEORY

For the diffusion model it was necessary to relate lightning currents flowing in the aircraft skin to voltages induced in aircraft electrical circuits inside. To derive this relationship, two fundamental laws were utilized:

- The Biot-Savart law, which describes the density of magnetic flux at a specific point away from a current carrying conductor.
- Faraday's law, which describes the voltage induced in a conductor by a changing magnetic flux passing through a loop formed by this conductor.

Any loop formed by an electrical conductor such as an aircraft electrical circuit, which is linked by a changing magnetic field, will have voltage induced in it equal to the negative time rate of change of the total magnetic flux linking the loop. This is Faraday's law and is expressed as:

$$e_m = -\frac{d\psi}{dt} \qquad (10)$$

where:

e_m = total emf (volts)

ψ = total flux (webers)

t = time (seconds)

It was next necessary to relate the total flux, Ψ, to the lightning current.

The magnetic flux which links an open surface such as that surrounded by an aircraft electrical circuit (including its return path) can be found by integrating the flux density, B, over the surface area linked by B. This may be expressed as

$$\psi = \int\int_s B \cdot ds \qquad (11)$$

where:

ψ = total flux (webers)

B = flux density (Wb/m^2)

S = surface area (m^2)

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Equations 10 and 11 relate induced voltage flux to total flux, and total flux to flux density. In relating flux density to lightning current it is appropriate to consider the physical situation which exists when lightning strikes an aircraft structure. Shown in Figure 8 is a filamentary representation of an aircraft wing, inside which is located an electrical conductor and its airframe return, forming the circuit loop ABCD. Because of the short time duration of most lightning strokes, nearly all of the lightning current flows in the skin rather than through internal spars and ribs; therefore only the skin is represented.

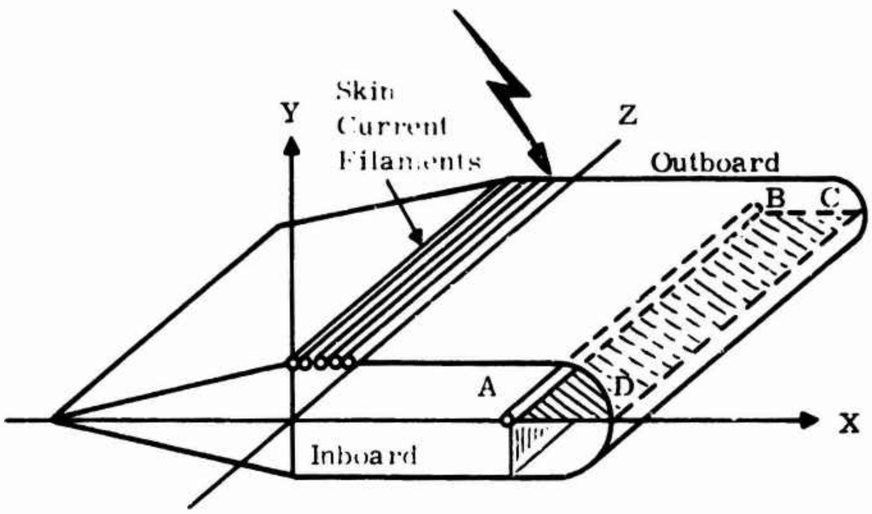


Figure 8. Circuit Wire in Aircraft Structure That Has Been Struck by Lightning

Assuming that this is so and that lightning current flows in a lineal direction, the aircraft structure can be represented by a large number, n, of parallel skin current filaments. The voltage, V_{A-D}, appearing at the inboard end and the outboard end of the loop is equal to the line integral of voltage induced around the loop ABCD. As previously discussed, the voltage induced in the loop is dependent upon the magnetic flux passing through this loop. This flux is in turn a function of flux density, as indicated by Equation 11. For the arrangement shown in Figure 9, the magnetic flux density produced

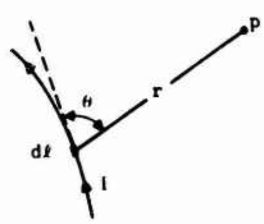


Figure 9. Current Carrying Filament

at some point, p, with respect to a current filament is defined by the Biot-Savart law as

$$B_n = \frac{\mu I_r}{4\pi} \int \frac{\sin\theta}{r^2}\, dl \tag{12}$$

where:

I_n = current (amperes)

B_n = magnetic flux density (Wb/m^2)

l, r = dimensions (meters)

μ = permeability of the medium (for air = $4\pi \times 10^{-7}$ H/m)

Note that at this point an expression for the flux density has been introduced which is dependent upon current. Since the structure has been represented by a parallel array of n current carrying filaments, there will be n contributions to the flux density at point p and all other such points in space.

It still remains to express the flux density B in terms of the airframe geometry. Figure 10 shows a typical skin current filament and the aircraft circuit loop previously considered. To obtain the total flux passing through the loop it is necessary to integrate the flux density over the loop area. Equation 12 is expressed in terms of the geometry of Figure 10. From this is obtained

$$B = \frac{\mu I}{4\pi} \int_e^{\ell} \frac{r}{\sqrt{(\ell - z)^2 + r^2}} \cdot \frac{1}{(\ell - z)^2 + r^2}\, dz \quad \text{1st integral (13)}$$

$$= + \frac{\mu_e I}{4\pi} \int \frac{r}{\sqrt{(z - \ell)^2 + r^2}} \cdot \frac{1}{(z - \ell)^2 + r^2}\, dz \quad \text{2nd integral (14)}$$

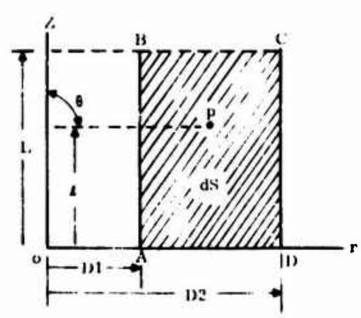


Figure 10. Skin Current Filament and Flux Density Through Aircraft Electrical Circuit Loop ABCD

The first integral (Equation 13) can be rewritten as Equation 15 and integrated by basic integral no. 173 (Ref. 8, p. 71) as follows:

$$1^{st}\text{ integral} = \frac{\mu I}{4\pi}\int_0^{\ell} \frac{r}{(\ell^2 + Z^2 - 2\ell Z + r^2)^{3/2}}\, dZ \tag{15}$$

$$= -\frac{\mu I}{4\pi}\left[\frac{2r(2Z - 2\ell)}{[4\ell^2 - 4(r^2 + \ell^2)]\sqrt{Z^2 - 2\ell Z + r^2 + \ell^2}}\right]_0^{\ell} \tag{16}$$

$$1^{st}\text{ integral} = \frac{\mu I}{4\pi}\left[\frac{(Z - \ell)}{r\sqrt{(Z - \ell)^2 + r^2}}\right]_0^{\ell} \tag{17}$$

where, for the basic integral no. 173 (Ref. 8),

$$a = 1,\ b = -2\ell \text{ and } c = (r^2 + \ell^2)$$

The second integral (Equation 14) can be rewritten and solved in the same manner as the first integral:

$$2^{nd}\text{ integral} = \frac{\mu I}{4\pi}\int_0^{L} \frac{r}{(Z^2 - 2\ell Z + \ell^2 + r^2)^{3/2}}\, dZ \tag{18}$$

$$= \frac{\mu I}{4\pi}\left[\frac{(Z - \ell)}{r\sqrt{(Z - \ell)^2 + r^2}}\right]_{\ell}^{L} \tag{19}$$

It is seen that the integral of Equation 17 is evaluated in the Z direction from the bottom of the filament at 0 to ℓ, and the integral of Equation 19 is evaluated from ℓ to the top of the filament at L. This integration gives B as a function of position in terms of ℓ and r:

$$B(\ell, r) = \frac{\mu I}{4\pi}\left[\frac{\ell}{r\sqrt{\ell^2 + r^2}} + \frac{(L - \ell)}{r\sqrt{(L - \ell)^2 + r^2}}\right] \tag{20}$$

Equation 20 is thus a general expression for the flux density, B, at a point at some distance from any of the current carrying filaments.

Now that an analytical expression has been developed for flux density, the flux linking the circuit loop can be determined by integrating the flux density in the manner suggested by Figure 11 and Equation 21:

$$\psi = \iint_S B \cdot ds \tag{21}$$

The circuit loop inside an aircraft structure need not run the entire length, L, of that structure. It may instead begin at any arbitrary point, ℓ_1, and terminate at any arbitrary point, ℓ_2. Therefore ℓ_1 is the lower limit and ℓ_2 is the upper limit of the first integration over the circuit loop length. The

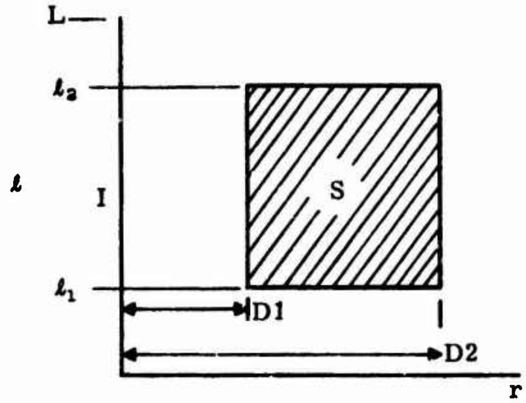


Figure 11. Skin Current Filament and Adjacent Circuit Loop

flux linking the loop is also dependent upon the distance to the loop location from the current filament. Thus the flux linking the loop in the radial direction is simply all of the flux out to a radial distance D_2 minus all of the flux out to a radial distance D_1. Equation 21 now has limits and can be expressed as

$$\psi = \int_{D_1}^{D_2} \int_{\ell_1}^{\ell_2} B \cdot dZ \cdot dr \tag{22}$$

Inserting the expression for B and performing the double integration yields the flux linking the circuit loop due to a single skin current filament. This expression is presented as Equations 23 through 26:

$$\psi = \frac{\mu_0 I}{4\pi} \Bigg[\left(\sqrt{\ell_2^2 + r^2} - \ell_2 \log_e\left(\frac{\sqrt{\ell_2^2 + r^2} + \ell_2}{r} \right) \right) \tag{23}$$

$$- \left(\sqrt{\ell_1^2 + r^2} - \ell_1 \log_e\left(\frac{\sqrt{\ell_1^2 + r^2} + \ell_1}{r} \right) \right) \tag{24}$$

$$+ \left(\sqrt{(\ell_1 - L)^2 + r^2} - (\ell_1 - L) \log_e\left(\frac{\sqrt{(\ell_1 - L)^2 + r^2} + (\ell_1 - L)}{r} \right) \right) \tag{25}$$

$$- \left(\sqrt{(\ell_2 - L)^2 + r^2} - (\ell_2 - L) \log_e\left(\frac{\sqrt{(\ell_2 - L)^2 + r^2} + (\ell_2 - L)}{r} \right) \right) \Bigg]_{D_1}^{D_2} \tag{26}$$

A cross-sectional view of the situation shown in Figure 11 might appear as shown in Figure 12. From this figure it is clear that the circuit loop need not be in the same plane as the skin current filament. Note that the time varying current that forms a part of Equations 23 through 26 makes the flux a time varying function, as required by Faraday's law (Equation 10).

The flux linking the circuit loop caused by the current filament can be calculated by assigning appropriate values to ℓ_1 and ℓ_2, computing the value of ψ when $r = D_2$, and then subtracting from this the value of ψ when $r = D_1$.

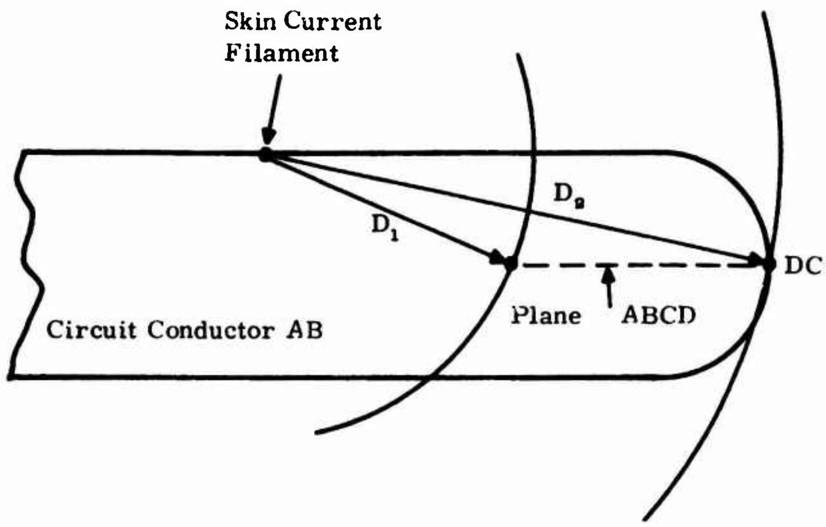


Figure 12. Cross-Sectional View of Wing Showing Distances Used to Compute Magnet Flux Passing Through Circuit Loop ABCD by Equations 23-26

The total flux, which links the loop due to all current filaments, is the summation of fluxes from all such filaments that pass through the same loop. Since all filaments will be at different distances D_1 and D_2 from the loop, the evaluation of Equations 23 through 26 must be performed n times. The transfer inductance, M, between a conductor carrying a current and another circuit through which flux generated by the first conductor passes is generally defined as

$$M \quad \frac{\psi_{Total}}{I_{Total}} \tag{27}$$

The total transfer inductance is therefore the sum of all of the fluxes Ψ_n for all filaments, divided by the total current responsible for that flux, or

$$M = \sum_{n=1}^{n=n} \frac{\psi_n}{I_{Total}} \tag{28}$$

This inductive transfer function, when inserted into Equation 9, enables expression of the magnetically induced voltage in an aircraft electrical circuit as a function of the lightning current.

SKIN CURRENT DISTRIBUTION THEORY

Experimental measurements of skin currents in aircraft (Ref. 6) have indicated that lightning currents do not, in fact, distribute evenly around the circumference of an airframe cross section. In all but uniformly symmetrical bodies (e.g., a cylinder) the current in each filament comprising the body will be somewhat different from the current in its neighbors. Accordingly, a subroutine was developed to calculate the amount of current flowing in each

of the skin current filaments comprising the airframe sections (Ref. 9). This subroutine, which is based on inductive current division, is described in the following paragraph.

CURRENT DIVISION

Figure 13 shows mutually coupled inductances through which current flows and voltage is developed. If there are two circuits (Figure 13a), then

$$V_1 = L_1 \left(\frac{d}{dt}\right) i_1 - M_{12} \left(\frac{d}{dt}\right) i_2 \tag{29}$$

$$V_2 = -M_{21} \left(\frac{d}{dt}\right) i_1 + L_2 \left(\frac{d}{dt}\right) i_2 \tag{30}$$

Only the bilateral case, in which $M_{12} = M_{21}$, will be treated here. This is no real restriction, because in all physically realizable systems mutual inductance is bilateral. Only the case in which all currents are in phase -- the low-frequency case -- will be treated. While in physical systems this need not be so, there are many systems in which current division is controlled only by inductive effects. Purely for ease of numerical analysis, only the frequency for which (d/dt) is numerically equal to unity will be considered. The analysis is valid for other frequencies, subject only to the above restrictions.

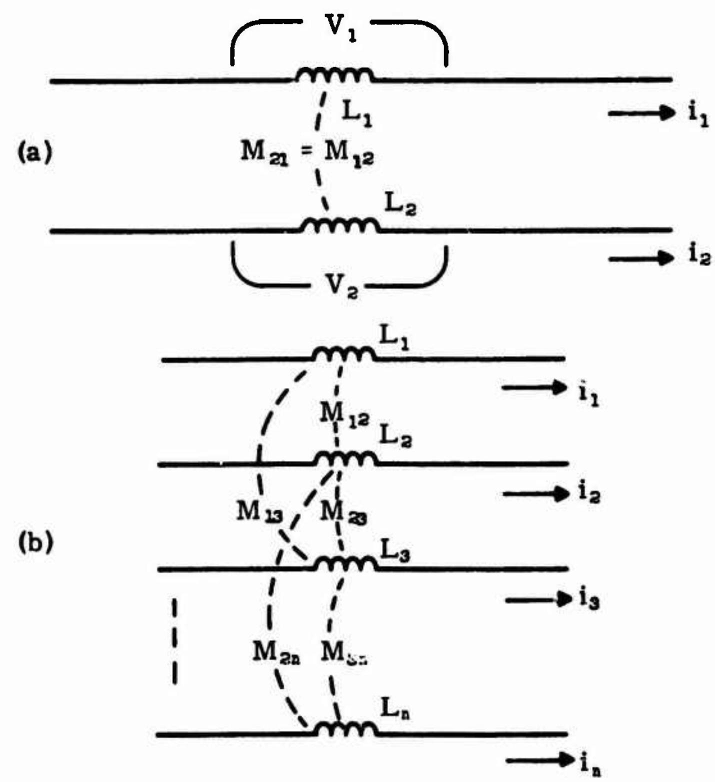


Figure 13. Mutually Coupled Inductances: a) Two Circuits, b) N Circuits

Under the above conditions,

$$V_1 = L_1 i_1 - M_{12} i_2 \qquad (31)$$

$$V_2 = -M_{21} i_1 + L_1 i_2 \qquad (32)$$

In the general case, Figure 13(b),

$$V_1 = L_1 i_1 - M_{12} i_2 - M_{13} i_3 \;....\; -M_{1n} i_n \qquad (33)$$

$$V_2 = -M_{21} i_1 + L_2 i_2 - M_{23} i_3 \;....\; -M_{2n} i_n \qquad (34)$$

$$V_3 = -M_{31} i_1 - M_{32} i_2 - L_3 i_3 \;....\; -M_{3n} i_n \qquad (35)$$

$$\vdots$$

$$V_n = -M_{n1} i_1 - M_{n2} i_2 - M_{n3} i_3 \;....\; L_n i_n \qquad (36)$$

Equations 33 through 36 may be placed in matrix notation as

$$\begin{vmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_n \end{vmatrix} = \begin{vmatrix} L_1 & -M_{12} & -M_{13} & .... & -M_{1n} \\ -M_{21}+ & L_2 & -M_{23} & .... & -M_{2n} \\ -M_{31} & -M_{32} & -L_3 & .... & -M_{3n} \\ \vdots & & & & \\ -M_{n1} & -M_{n2} & -M_{n3} & .... & -L_{nn} \end{vmatrix} \times \begin{vmatrix} i_1 \\ i_2 \\ i_3 \\ \vdots \\ i_n \end{vmatrix} \qquad (37)$$

or, in more compact notation:

$$| V | = | M | \times | i | \qquad (38)$$

Multiplying by the inverse of the M matrix, $|M|^{-1}$:

$$| M |^{-1} \times | V | = | M |^{-1} \times | M | \times | i | \qquad (39)$$

or:

$$| i | = | M |^{-1} \times | V | \qquad (40)$$

$$\begin{vmatrix} i_1 \\ i_2 \\ i_3 \\ \vdots \\ i_n \end{vmatrix} = \begin{vmatrix} m_{11} & m_{12} & m_{13} & .... & m_{1n} \\ m_{21} & m_{22} & m_{23} & .... & m_{2n} \\ m_{31} & m_{32} & m_{33} & .... & m_{3n} \\ \vdots & & & & \\ m_{n1} & m_{n2} & m_{n3} & .... & m_{nn} \end{vmatrix} \times \begin{vmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_n \end{vmatrix} \qquad (41)$$

where m_{11}, m_{12}, m_{13} .... are the elements of the inverse of the M matrix.

If all of the voltages are the same and equal to V, as in the case if all of the inductances are connected in parallel, the absolute current in each element is

$$i_1 = (m_{11} + m_{12} + m_{13} \;....\; + m_{1n})V \qquad (42)$$

$$i_2 = (m_{21} + m_{22} + m_{23} \;....\; + m_{2n})V \qquad (43)$$

$$i_3 = (m_{31} + m_{32} + m_{33} \dots + m_{3n})V \tag{44}$$

$\vdots$

$$i_n = (m_{n1} + m_{n2} + m_{n3} \dots + m_{nn})V \tag{45}$$

The total current that flows, which is proportional to the impressed voltage, is

$$i_T = (i_1 + i_2 + i_3 + \dots i_n)V \tag{46}$$

The fraction of the total current that flows in each circuit is

$$I_1 = \frac{i_1}{i_T} \tag{47}$$

$$I_2 = \frac{i_2}{i_T} \tag{48}$$

$$I_3 = \frac{i_3}{i_T} \tag{49}$$

$\vdots$

$$I_n = \frac{i_n}{i_T} \tag{50}$$

SELF AND MUTUAL INDUCTANCES

This analysis treats the case in which the self and mutual inductances are those of parallel circular conductors of a sufficient length, compared to the spacing between conductors, that all end effects may be ignored. Only the case in which all conductors are far removed from any conducting surfaces such as ground will be considered.

Figure 14 shows a single conductor in space, carrying a current, I. The magnetic field intensity in the space around this conductor is

$$H = \frac{I}{2\pi r} \text{ A/m} \tag{51}$$

The magnetic flux density is

$$\begin{aligned} B = \mu H &= \frac{4\pi \times 10^{-7} I}{2\pi r} \\ &= 2 \times 10^{-7} \frac{I}{r} \text{ Wb/m}^2 \end{aligned} \tag{52}$$

The self-inductance of the conductor, i, is defined as

$$L_i = \frac{\Delta\varphi}{\Delta I_i} \text{ Wb/A} \tag{53}$$

The ratio of webers per ampere is, of course, given the name "henries." The magnetic flux, φ, is equal to the area under the B curve (Figure 14) from r_1 (the conductor surface) out to some other point, R, which defines the return

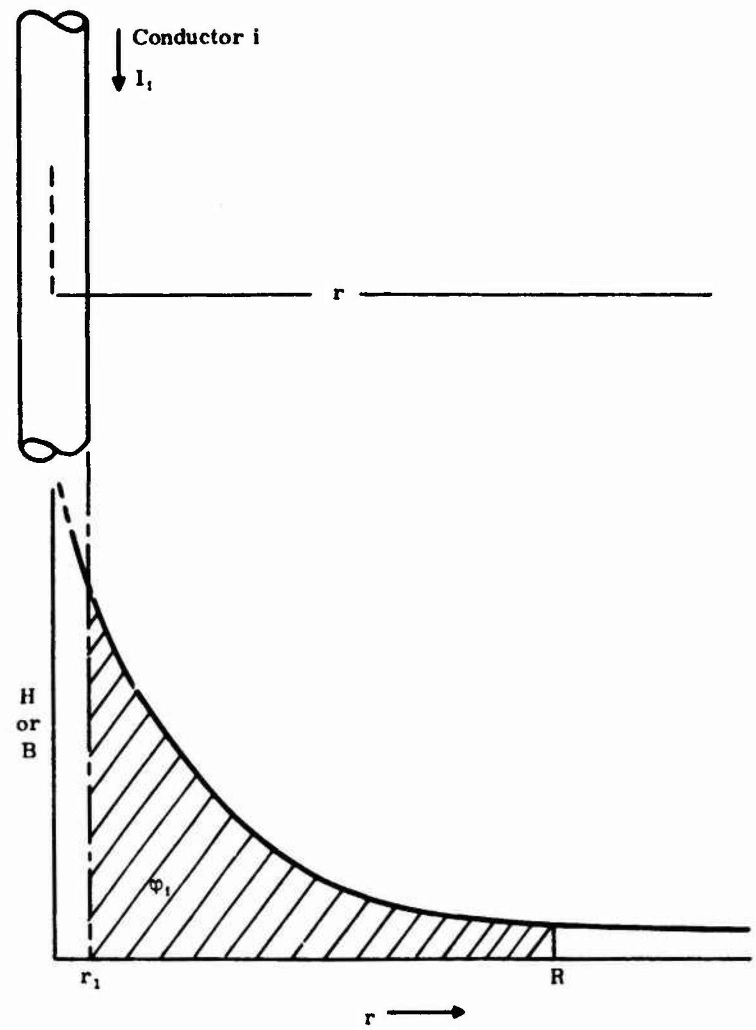


Figure 14. Self-Inductance

path for the current, I, in the conductor. If one postulates a conductor carrying direct current and located in free space, this return path will be at infinity. The flux density goes to zero as R goes to infinity, but the area under the curve, φ, also goes to infinity. If φ goes to infinity, then L, as defined in Equation 53, also goes to infinity. Accordingly, one cannot speak of a single value as describing the inductance of an isolated conductor.

If the conductor is carrying a transient or alternating current rather than a direct current, an inductance can be defined; this is because the magnetic fields cannot instantaneously fill the entire region around the conductor but, instead, propagate outward at the speed of light. Because the effective distance to which they propagate is time or frequency dependent the inductance will also be time or frequency dependent. In this analysis, R is taken as the distance to which a field could propagate in one microsecond -- 300 meters.

The area, φ, under the B curve of Figure 14 is

$$\varphi = 2 \times 10^{-7} I \int_{r_1}^{R} \frac{dr}{r} \tag{54}$$

$$\varphi = 2 \times 10^{-7} I \log_{\epsilon} r \Big]_{r_1}^{R} \tag{55}$$

$$\varphi = 2 \times 10^{-7} I \log_{\epsilon} \frac{R}{r_1} \tag{56}$$

Remembering the definition of L (Equation 53),

$$L_i = \frac{\varphi_i}{I_i} = 2 \times 10^{-7} \log_{\epsilon} \frac{R}{r_1} \tag{57}$$

The mutual inductance between conductors i and j is defined as

$$M_{ij} = \frac{\Delta\varphi_j}{\Delta I_i} \tag{58}$$

φ_j, the flux linking conductor j and set up by the current I_i in conductor i (as shown in Figure 15), is

$$\varphi_j = 2 \times 10^{-7} I_i \int_{r_2}^{R} \frac{dr}{r} \tag{59}$$

$$\varphi_j = 2 \times 10^{-7} I_i \log_{\epsilon} \frac{R}{r_2} \tag{60}$$

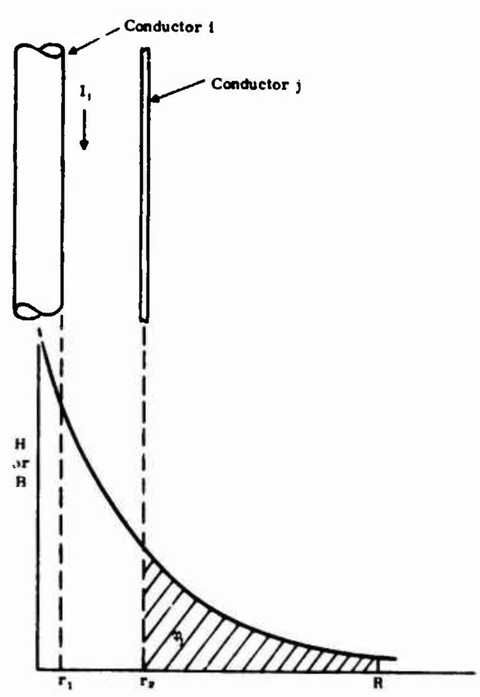


Figure 15. Mutual Inductance

Hence,

$$M_{ij} = 2 \times 10^{-7} \log_\epsilon \frac{R}{r_2} \tag{61}$$

PROGRAM OPERATION

With the cross section of the airframe in the X-Y plane, the X, Y, Z coordinates of the conductors, and their radii, are read and stored in a matrix, printed for inspection, and reread. The arbitrary distance to which the fields propagate (300 meters) is given as R5 in the computer listing. The spacing between all conductors is then calculated, and the mutual inductances are calculated and loaded in the array. Self-inductances are loaded into the appropriate elements, those on the main diagonal.

At this stage the matrix holds the absolute currents in the individual conductors (assuming V = 0), currents corresponding to those given in Equations 42 through 45. The total current is then calculated; the fractional current is then calculated and stored.

COMPUTER PROGRAM DIFFUSION

GENERAL DESCRIPTION

The computer program DIFFUSION was established to represent an aircraft as a combination of several independent sections. Each of these sections is represented in the computer program by an array of parallel current carrying filaments. Figure 16 shows a complete aircraft, while Figures 17 through 20 show the individual sections of the aircraft modeled by this program.

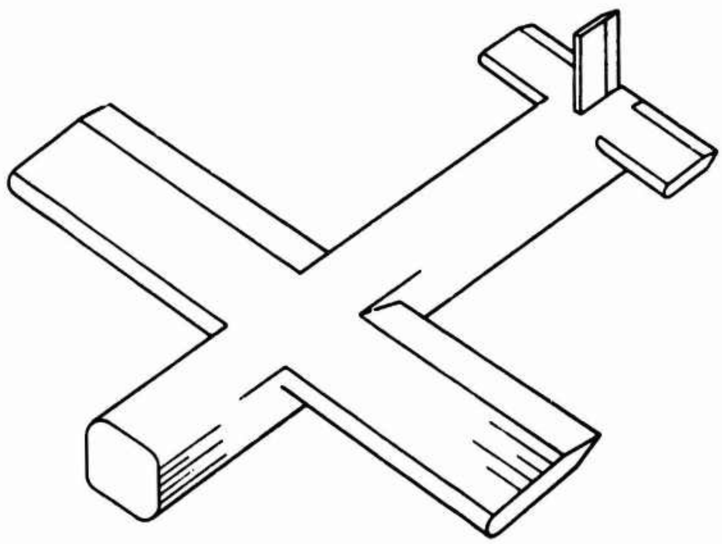


Figure 16. Complete Aircraft Represented by Parallel Current Carrying Filaments

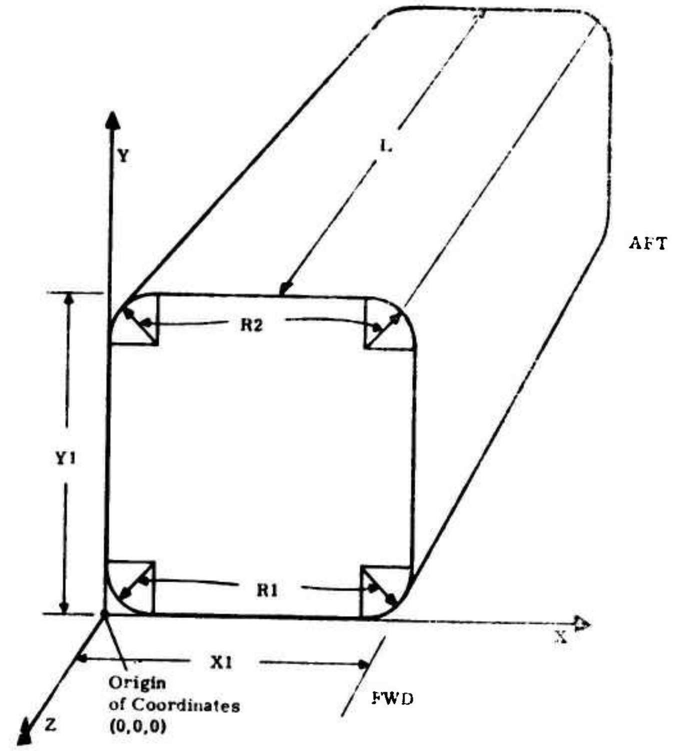


Figure 17. Fuselage Section

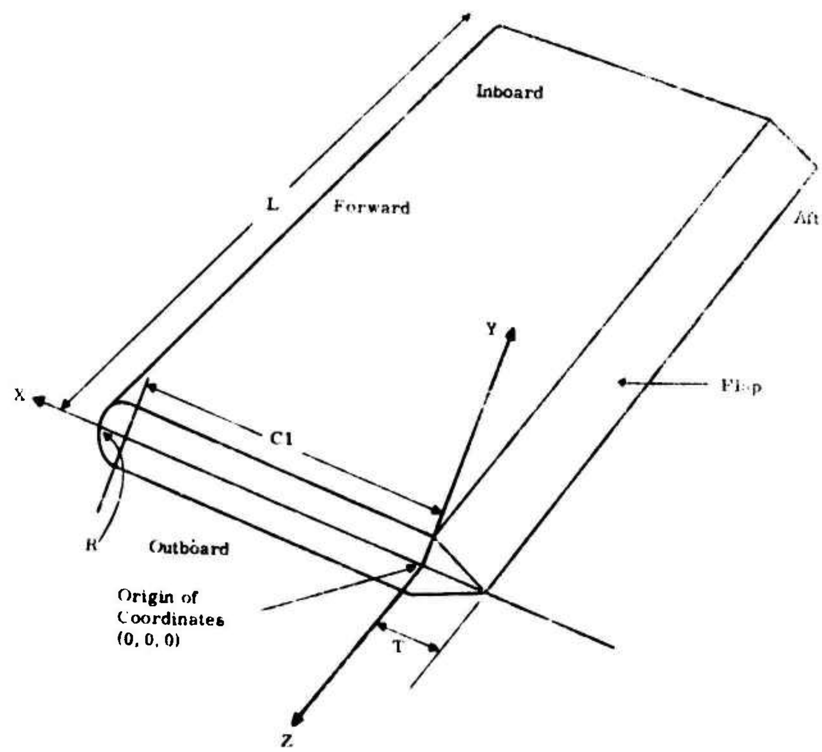


Figure 18. Wing Section

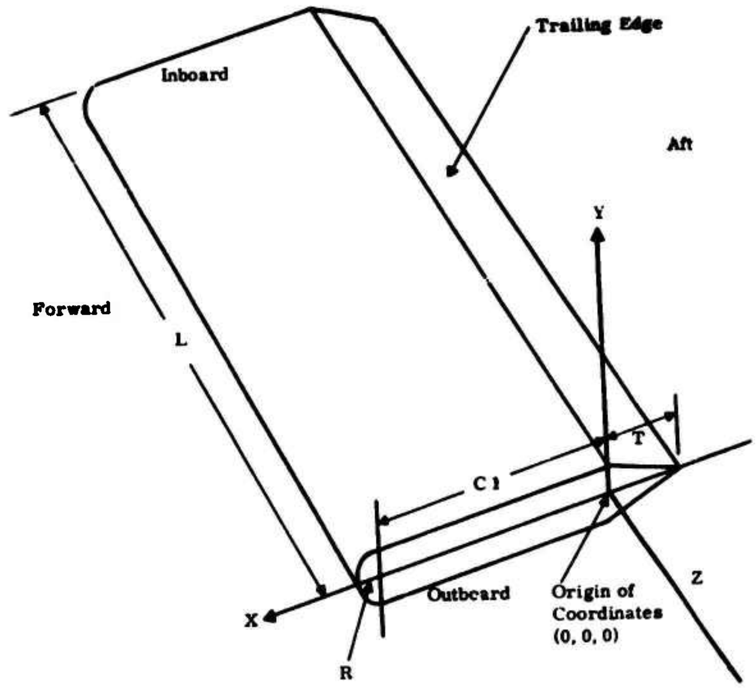


Figure 19. Horizontal Stabilizer Section

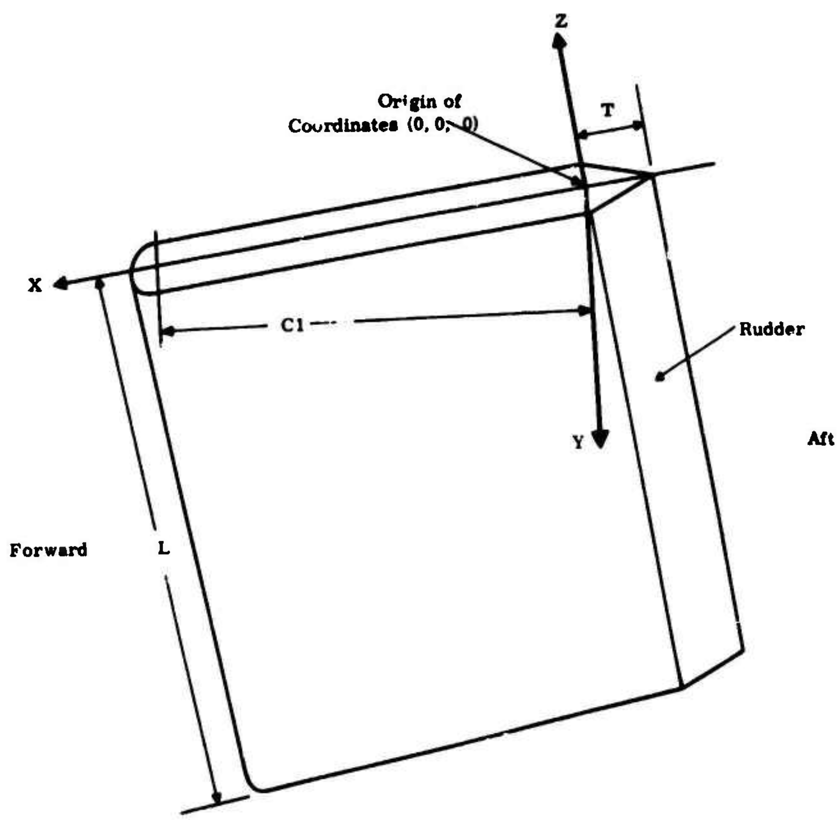


Figure 20. Vertical Stabilizer Section

Each section of the aircraft is completely described by several geometric constants, from which the computer program calculates the location of each current filament with respect to a coordinate system. Individual sections are shown in Figures 17 through 20.

The geometrical dimensions X1, R1, Y1, and Y2, etc. are read into the computer program as the first step in execution. At the same time, the initial location of an enclosed electrical circuit conductor and a set of modifiers are read in. These modifiers allow the user to reposition the electrical conductor during program execution.

The variations which are made under program control are enumerated and illustrated below, using a fuselage section as an example:

1. The X coordinate (Figure 21) may be varied horizontally in a stepwise manner from X_i (initial X coordinate) to X_f (final X coordinate).

2. The Y coordinate may be varied vertically in a stepwise manner from Y_i (initial Y coordinate) to Y_f (final Y coordinate) (Figure 22).

3. The length or position of the circuit conductor may be varied horizontally in a stepwise manner by varying the Z coordinate of either or both of the conductor end points, Z_1 and Z_2 (Figure 23).

4. Any combination of the X, Y, and Z coordinate variations may also be made. The X coordinate may be varied until it reaches a particular value (1 → 2), after which the Y coordinate may be varied (2 → 3) until it reaches a particular value; then the Z_1 and/or Z_2 coordinates may be varied until a final position/length is achieved (3 → 4) (Figure 24).

Incrementing of all three coordinates may occur sequentially, simultaneously, or in combination. Thus variation of one variable need not be terminated prior to changing the value of another of the variables (see Figure 25).

For each circuit conductor location the program then determines the magnetic flux density at the forward or inboard end of the circuit conductor as shown in Figure 26. It then computes the transfer inductance between the circuit formed by the enclosed conductor and airframe return and the current filaments used to represent the aircraft section under investigation.

Once the transfer inductance and resistance values have been computed, the open circuit voltage versus time is tabulated for Equation 62:

$$e_{oc} = R_T i(t) + M\frac{d(1 - e^{-\alpha t})\, i(t)}{dt} \tag{62}$$

The user may input as many different sets of data cards as are desired. The program will execute each set over the range of values given and print out the data generated for each set.

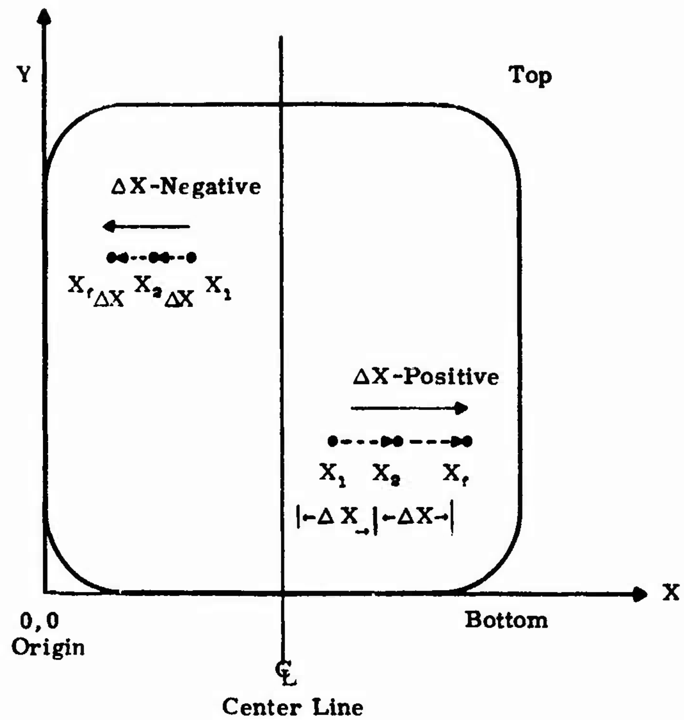


Figure 21. Permissible Variation of X Coordinate of Enclosed Electrical Circuit Conductor

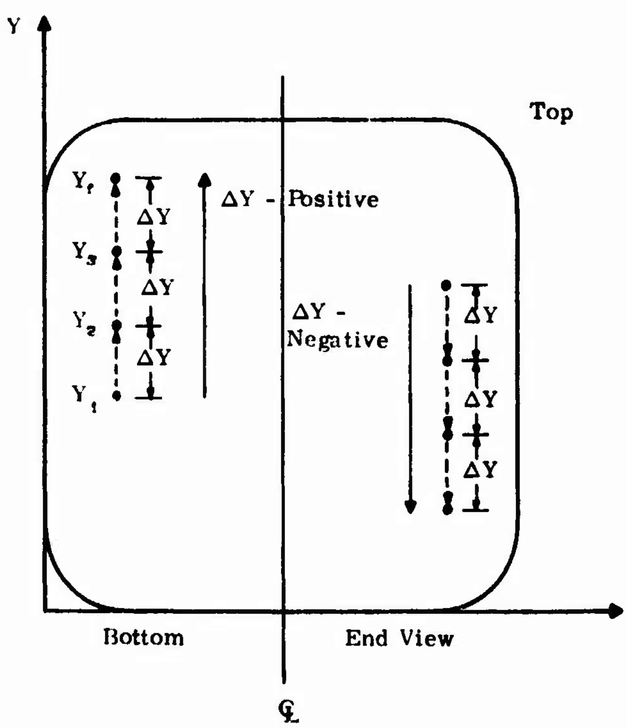


Figure 22. Permissible Variation of Y Coordinate of Enclosed Electrical Circuit Conductor

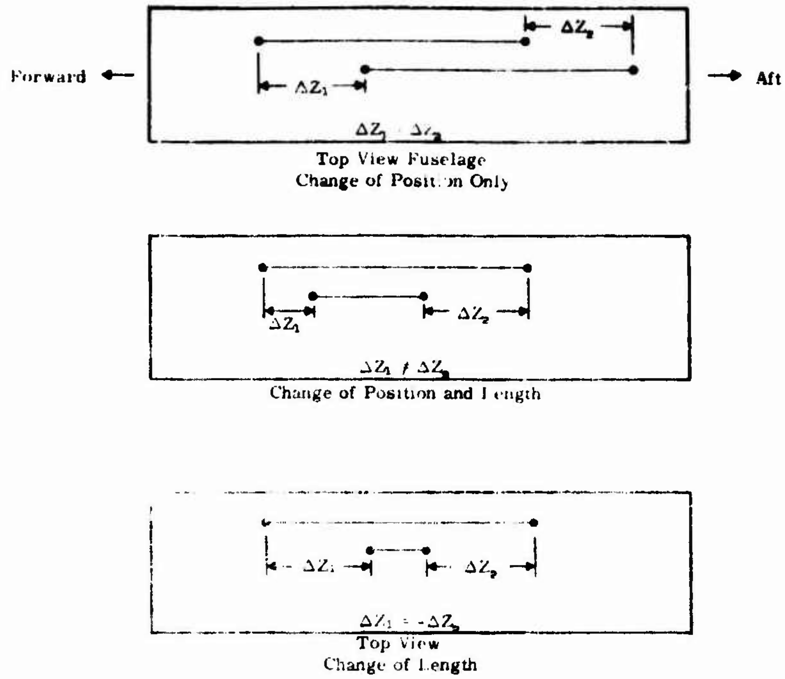


Figure 23. Permissible Variation of Z Coordinates of Enclosed Electrical Conductor

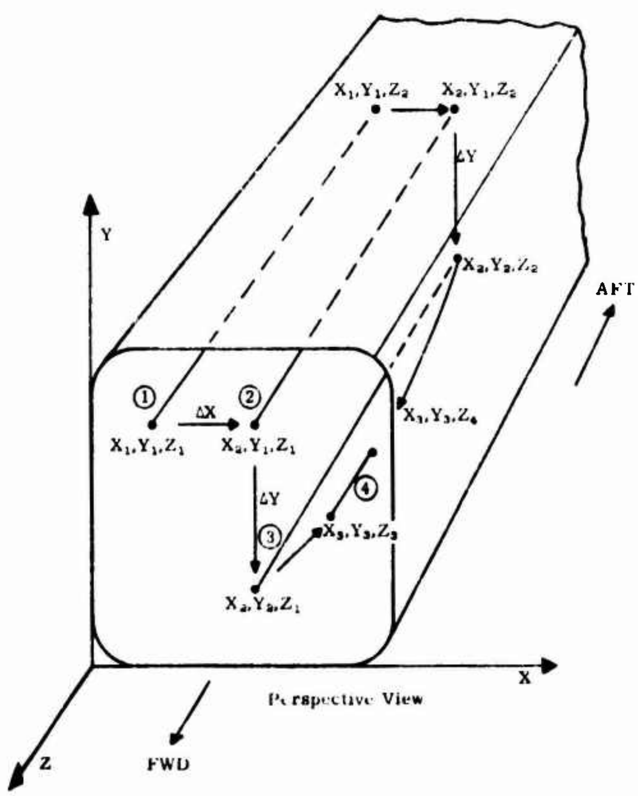


Figure 24. Permissible Variation of Enclosed Electrical Conductor Coordinates

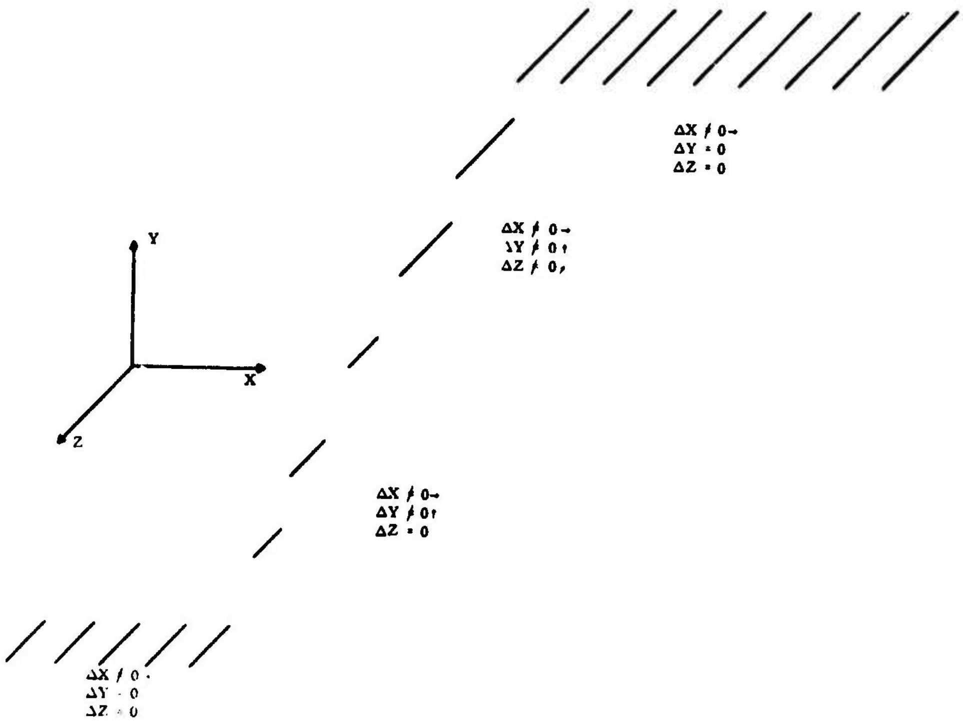


Figure 25. Example of a Possible Set of Variations of Circuit Conductor Location and Length

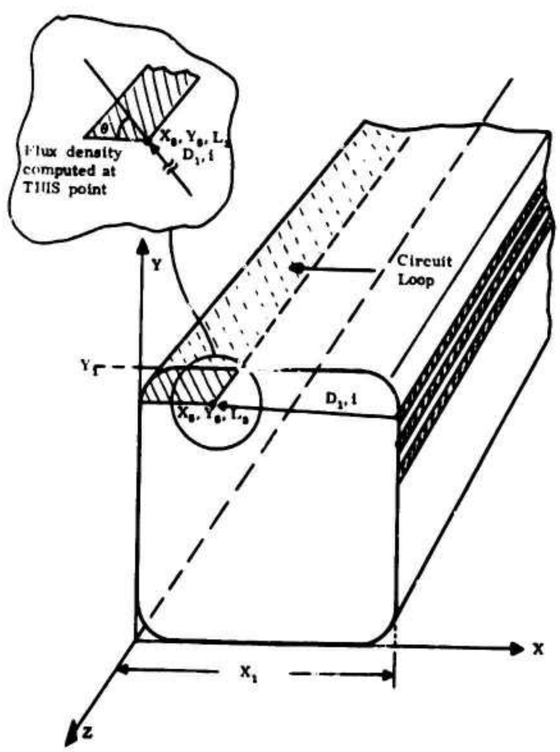


Figure 26. Location of Enclosed Circuit Loop and Flux Density Computation

The computer program initially divides the continuous geometrical structure into an array of parallel current carrying filaments and computes the current distribution in each filament, using the method already described under "Skin Current Distribution Theory." It then defines a horizontal plane defined by the circuit conductor and a return conductor in the airframe skin. The program computes the flux density, B, at the forward end of the circuit conductor (Figure 26) and the flux passing through the defined plane contributed by each of the current filaments.

These flux linkages are summed and divided by the total lightning current, to obtain the transfer inductance, M. If the location of the circuit conductor is to be repositioned, for design optimization studies, the computer program input data establish the step size and direction in which to move the electrical circuit conductor for the second operation. In such a case, new coordinates of another horizontal plane are determined and the flux density and flux computations are performed again. Each time the operation is performed, a flux density is determined at the new location of the forward end of the enclosed electrical conductor, as well as the total flux linking the newly defined circuit. After each set of conditions has been calculated the program determines whether there are other geometries to be evaluated.

DIFFUSION FLOW DIAGRAMS

An elementary flow diagram of the DIFFUSION computer program is shown here as Figure 27; Figure 28 is a detailed flow diagram of the program. A program listing for DIFFUSION is given in Figure 29*; the listing includes, in addition to the main portion, subroutines MATRIX, MATINV, and MATZER. The program begins (lines 1-103) with some introductory comments to assist the user in operation. It next establishes five files in which to temporarily store data generated by the program.

In lines 170-210 the program initializes the constants to be used in the computation and reads in a control variable, A, which routes the program to line 240, 1840, 1860, or 1880, depending on the geometry specified by the user. Upon selection of the appropriate geometry, the computer program reads in the data pertinent to that geometry, and then prints out a heading to indicate that diffusion coupling is being computed in the particular geometry named. The initial data read contains the location of the circuit conductor to be evaluated and a set of modifiers with which the user may change the position of the electrical circuit conductor inside the particular geometry. The user may make as many modifications in these data as he desires; for each modification, one program execution occurs.

After the circuit conductor location has been defined, the geometry that has been selected can be described as an array of current carrying filaments whose locations are computed from the constants of geometry and the mathematical expressions derived to analytically define that geometry. This is done in lines 270-1371 for a fuselage, or lines 187-2580 for wing type geom-

*This listing is for a program that will be run on the General Electric Time sharing computer. A program listing for the CDC6600 machine is included in Appendix III, "Program Listings for CDC6600 Computer."

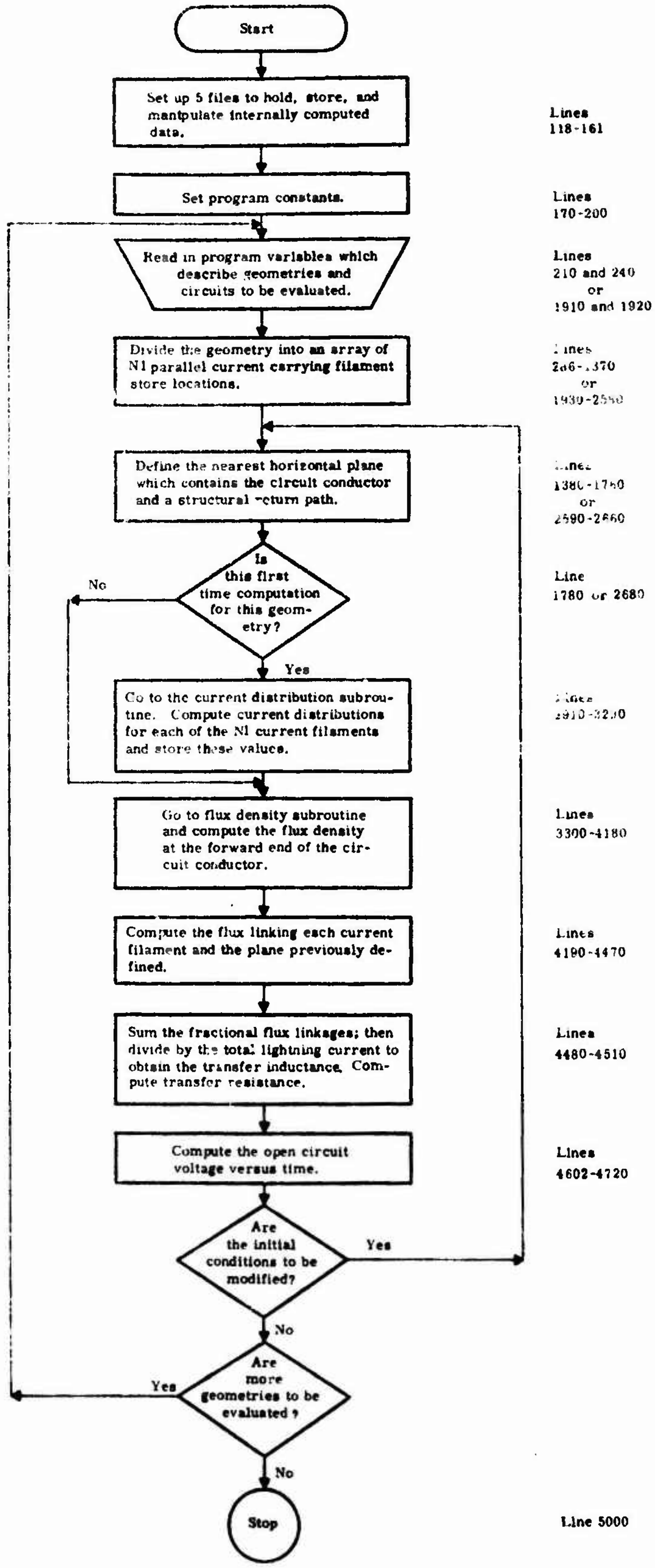


Figure 27. Elementary Flow Diagram of Diffusion Program

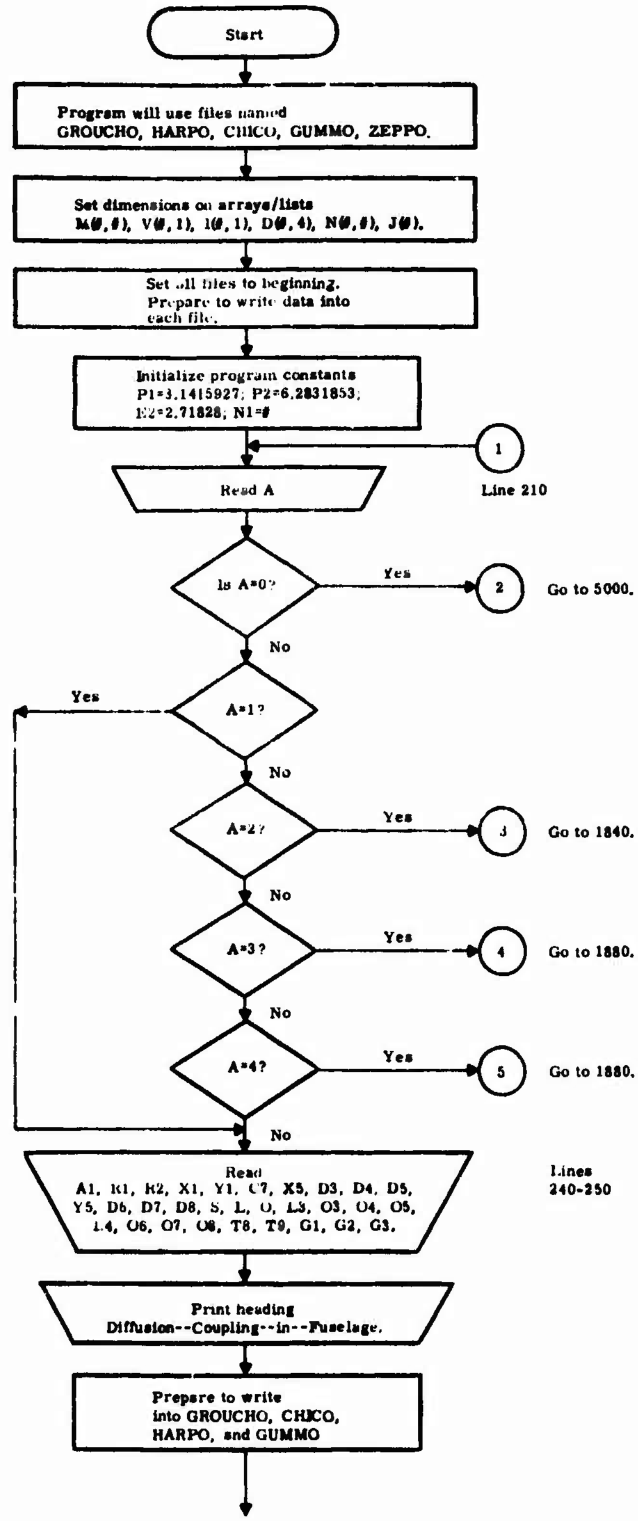


Figure 28. Detailed Flow Diagram of Diffusion Program (Sheet 1 of 23)

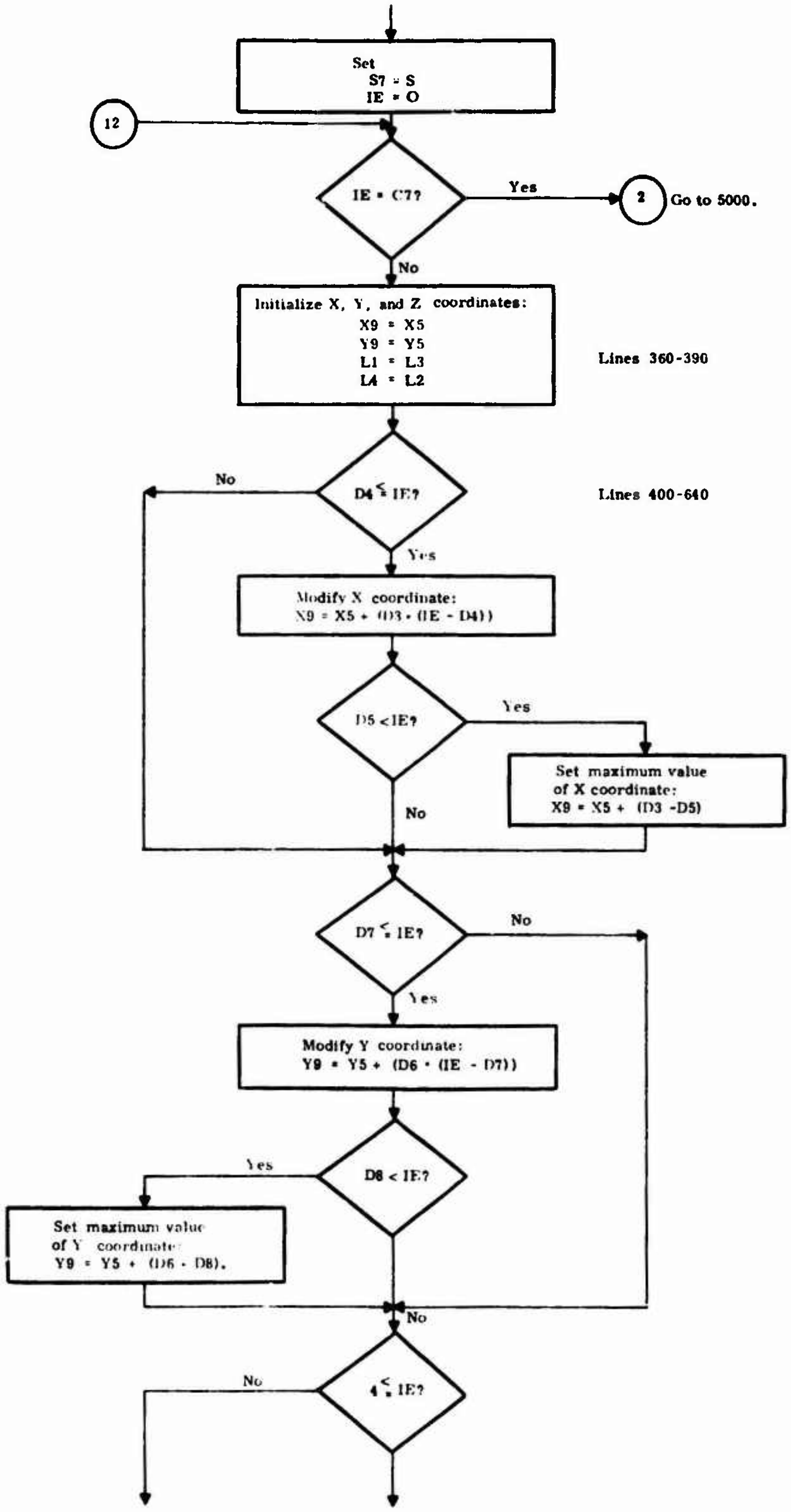


Figure 28. Detailed Flow Diagram (Sheet 2 of 23)

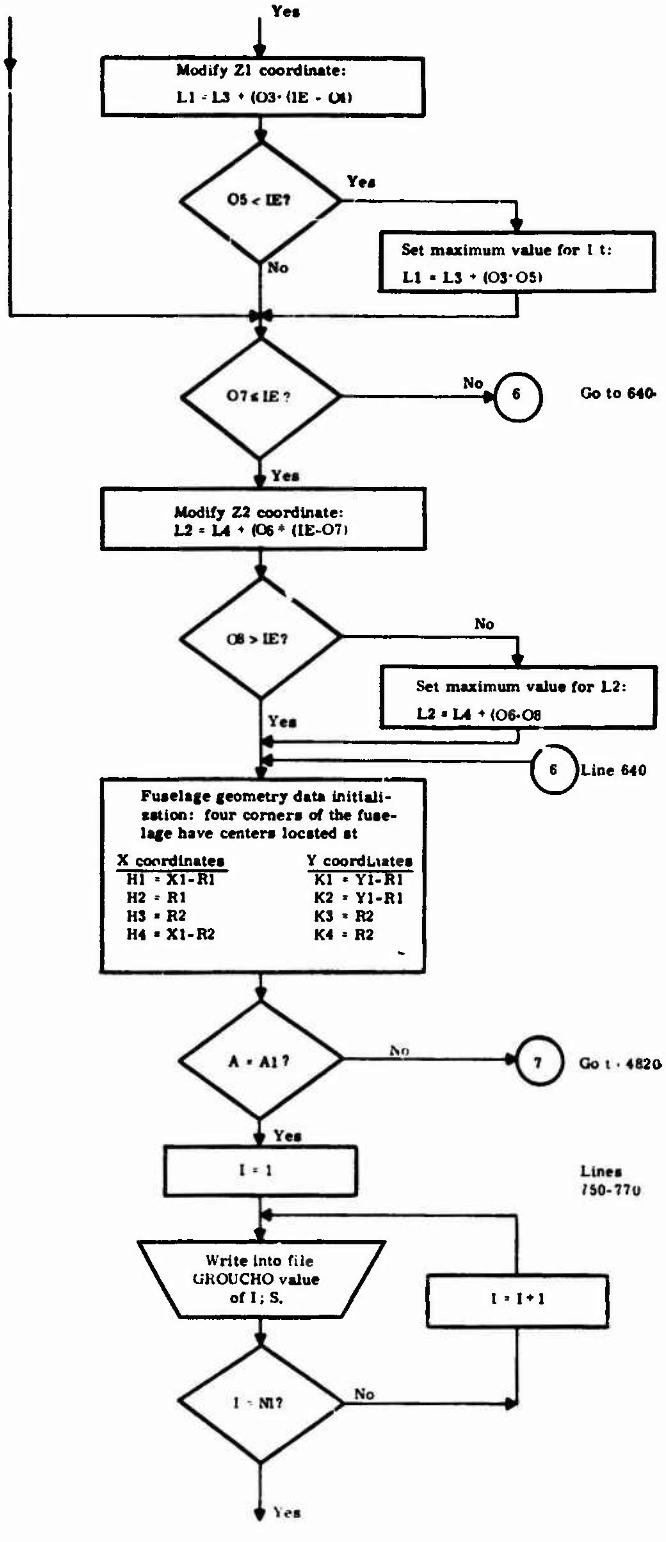


Figure 28. Detailed Flow Diagram (Sheet 3 of 23)

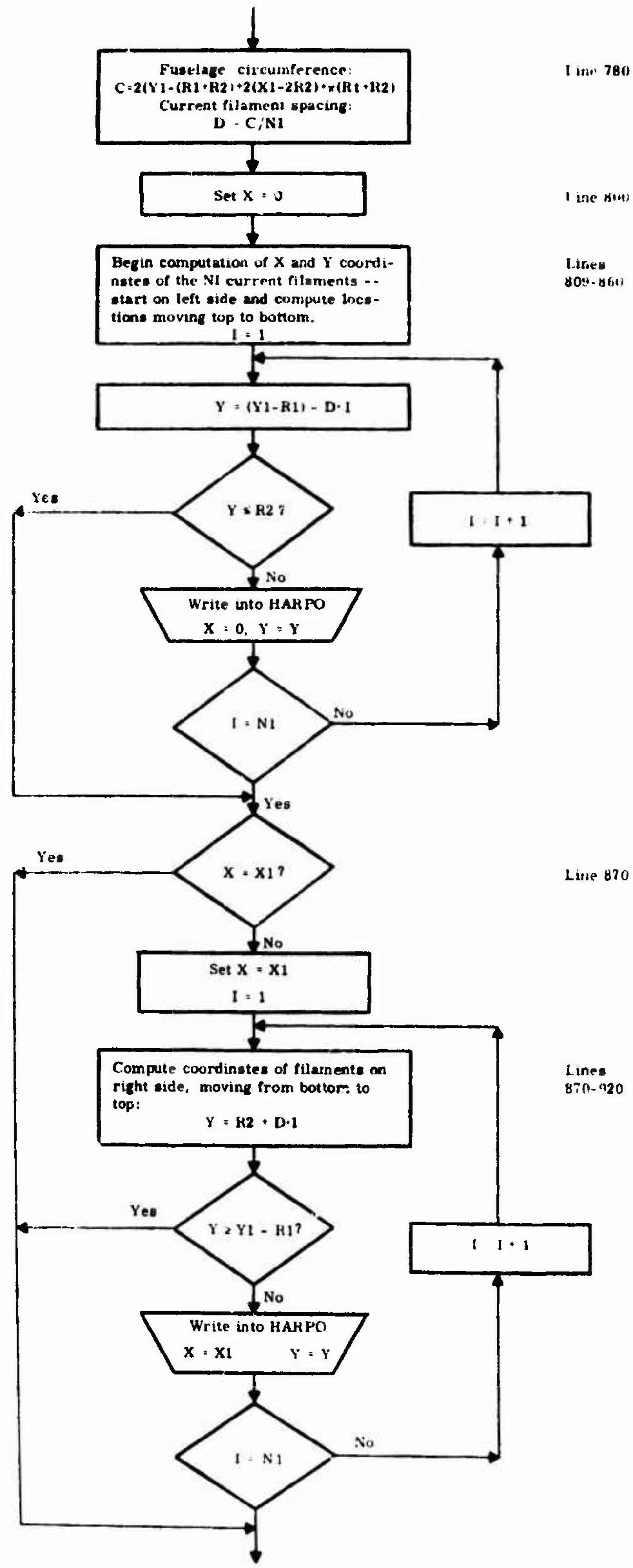


Figure 28. Detailed Flow Diagram (Sheet 4 of 23)

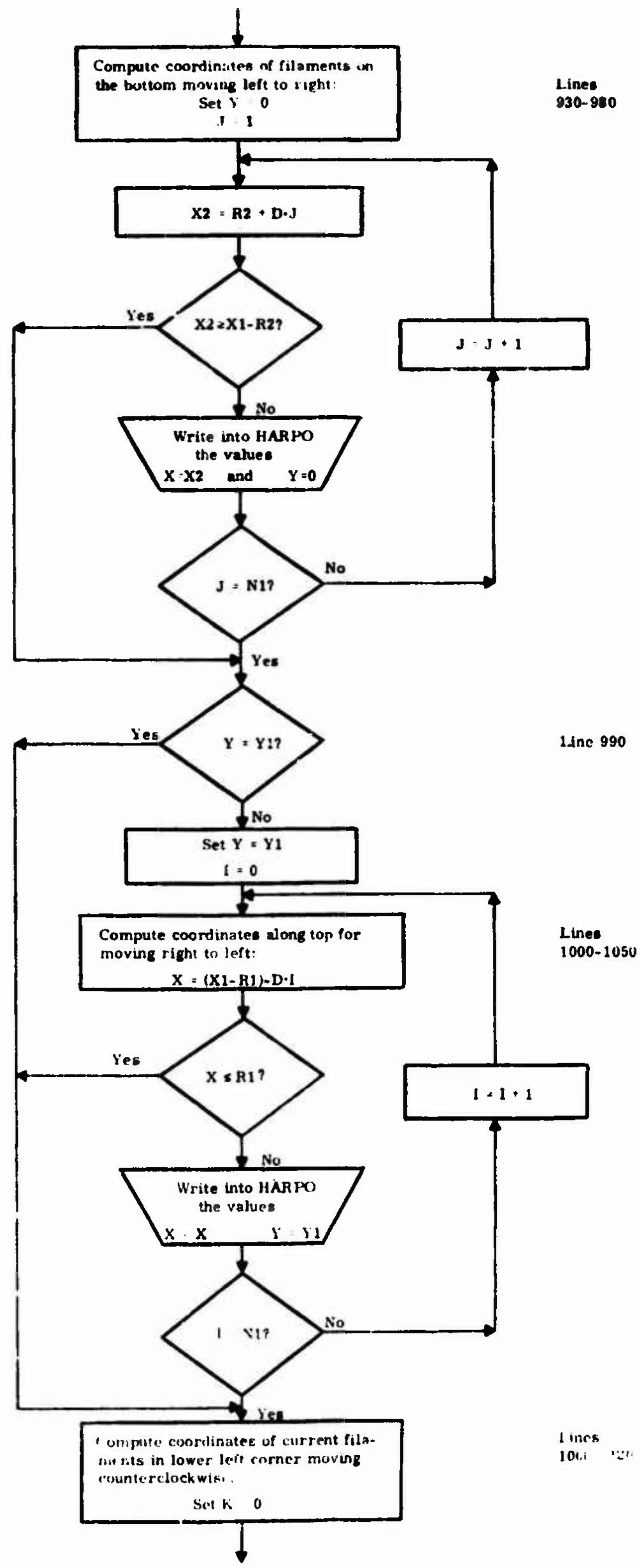


Figure 28. Detailed Flow Diagram (Sheet 5 of 23)

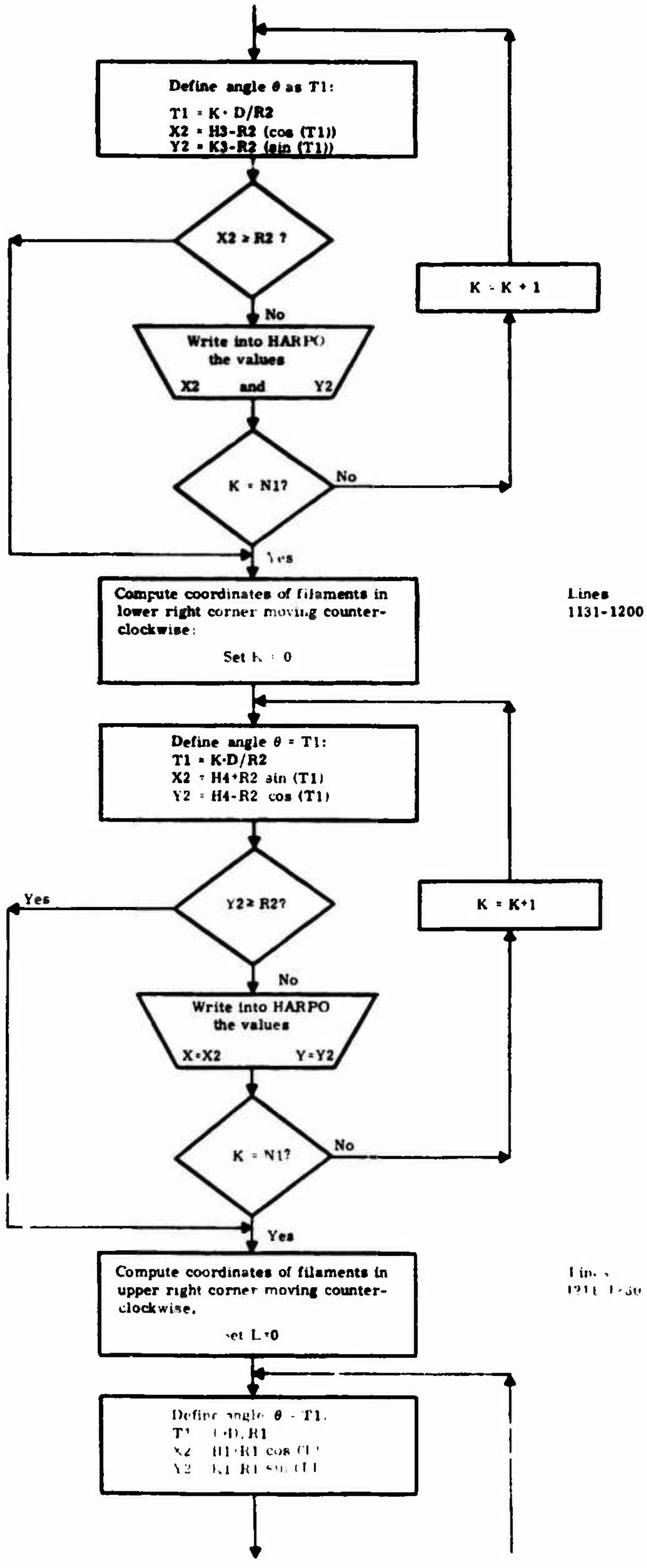


Figure 28. Detailed Flow Diagram (Sheet 6 of 23)

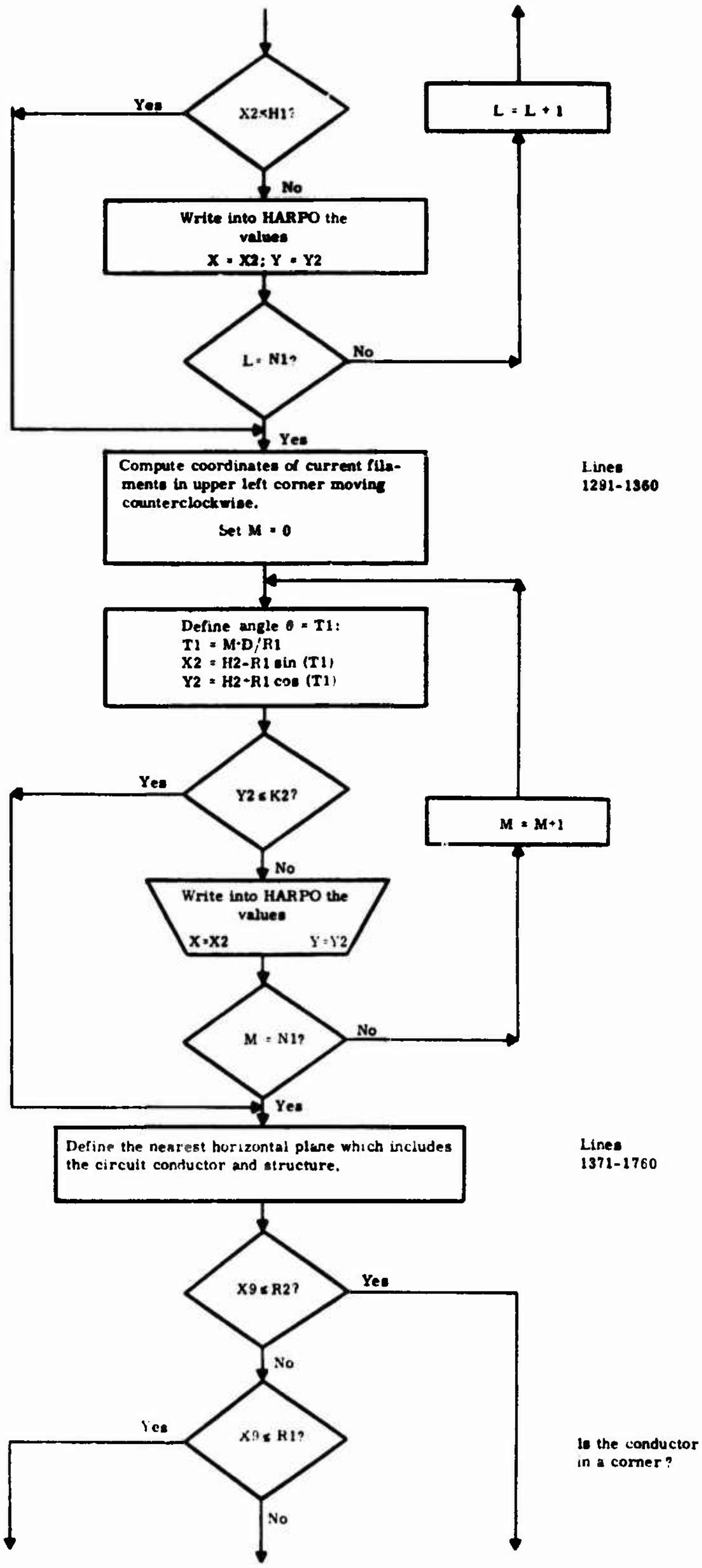


Figure 28. Detailed Flow Diagram (Sheet 7 of 23)

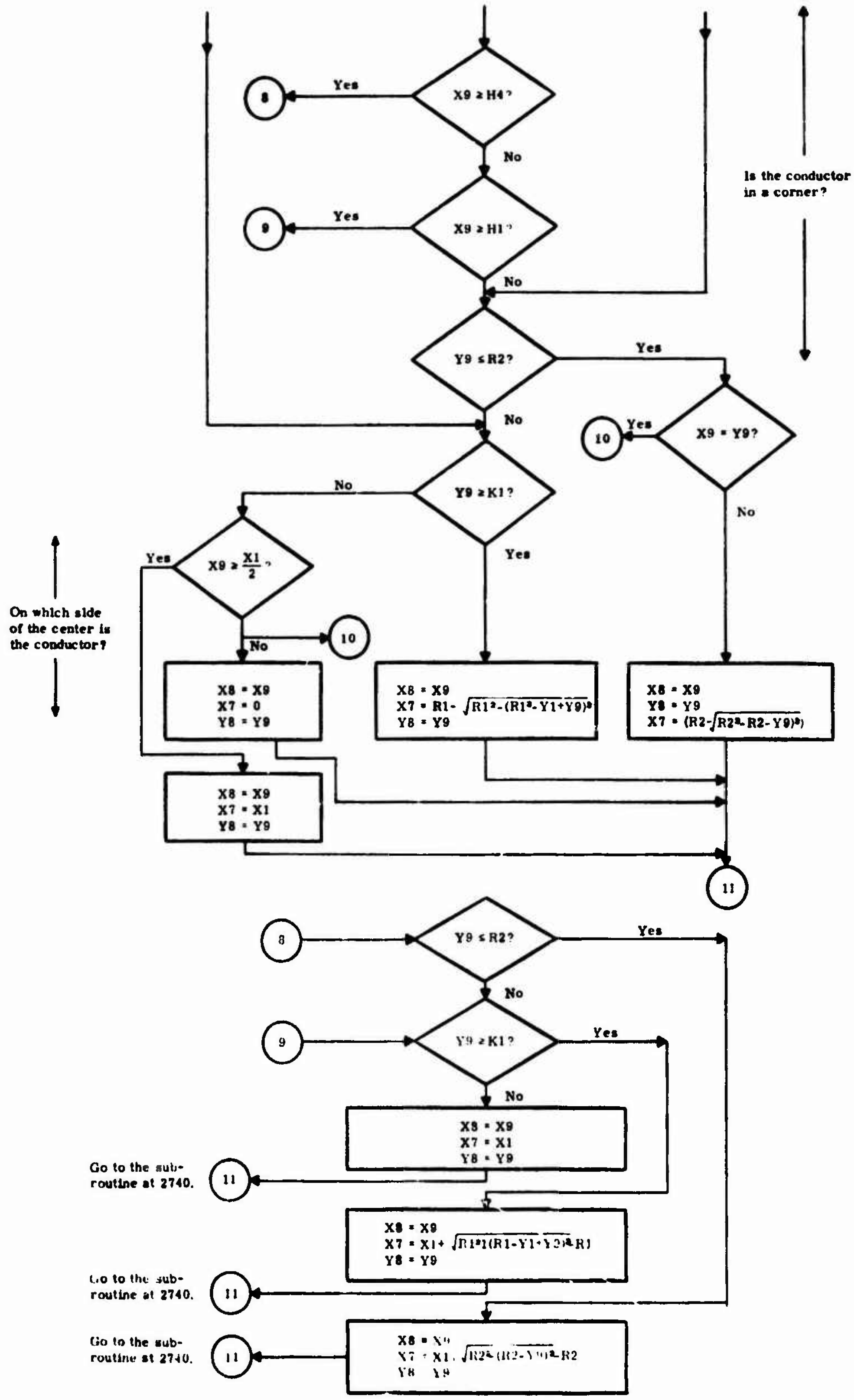


Figure 28. Detailed Flow Diagram (Sheet 8 of 23)

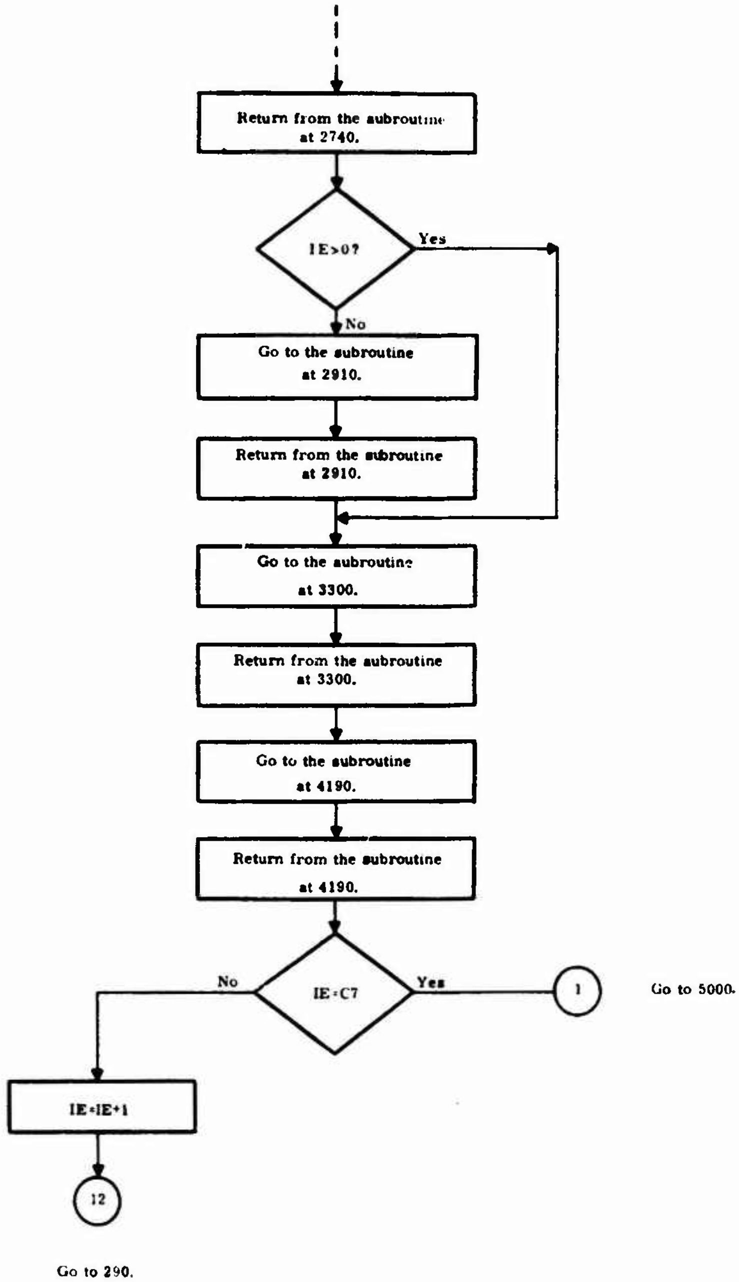


Figure 28. Detailed Flow Diagram (Sheet 9 of 23)

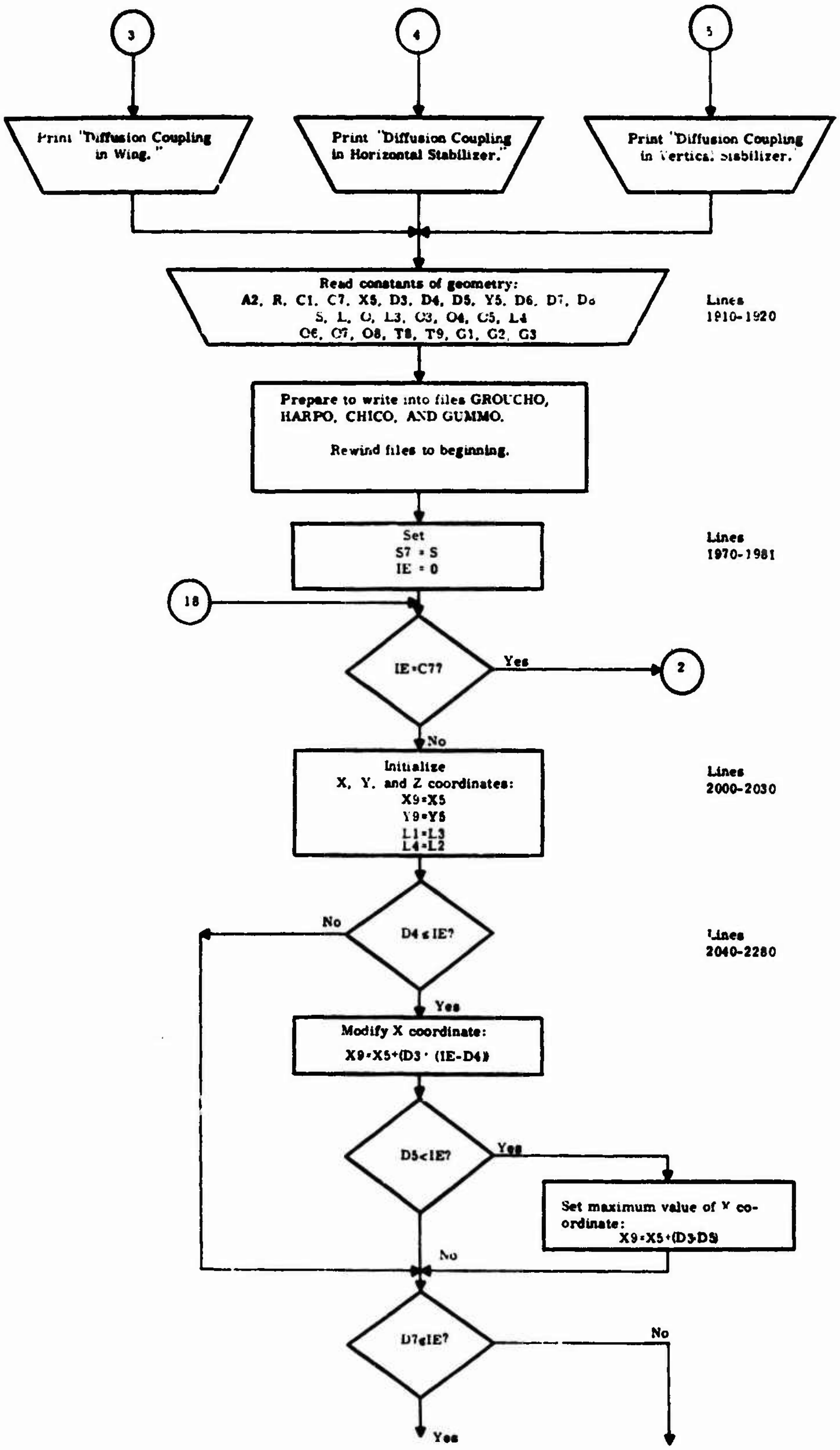


Figure 28. Detailed Flow Diagram (Sheet 10 of 23)

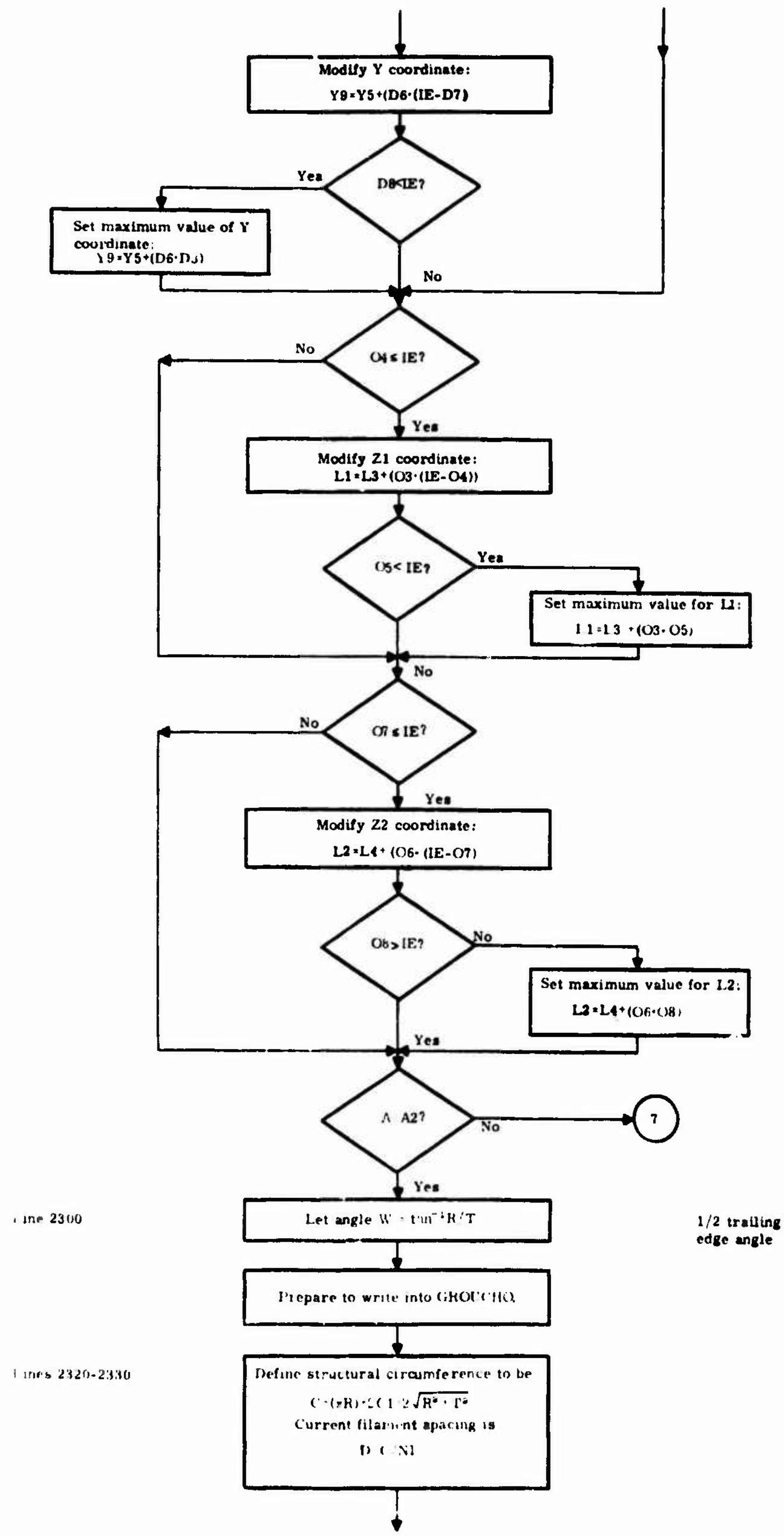


Figure 28. Detailed Flow Diagram (Sheet 11 of 23)

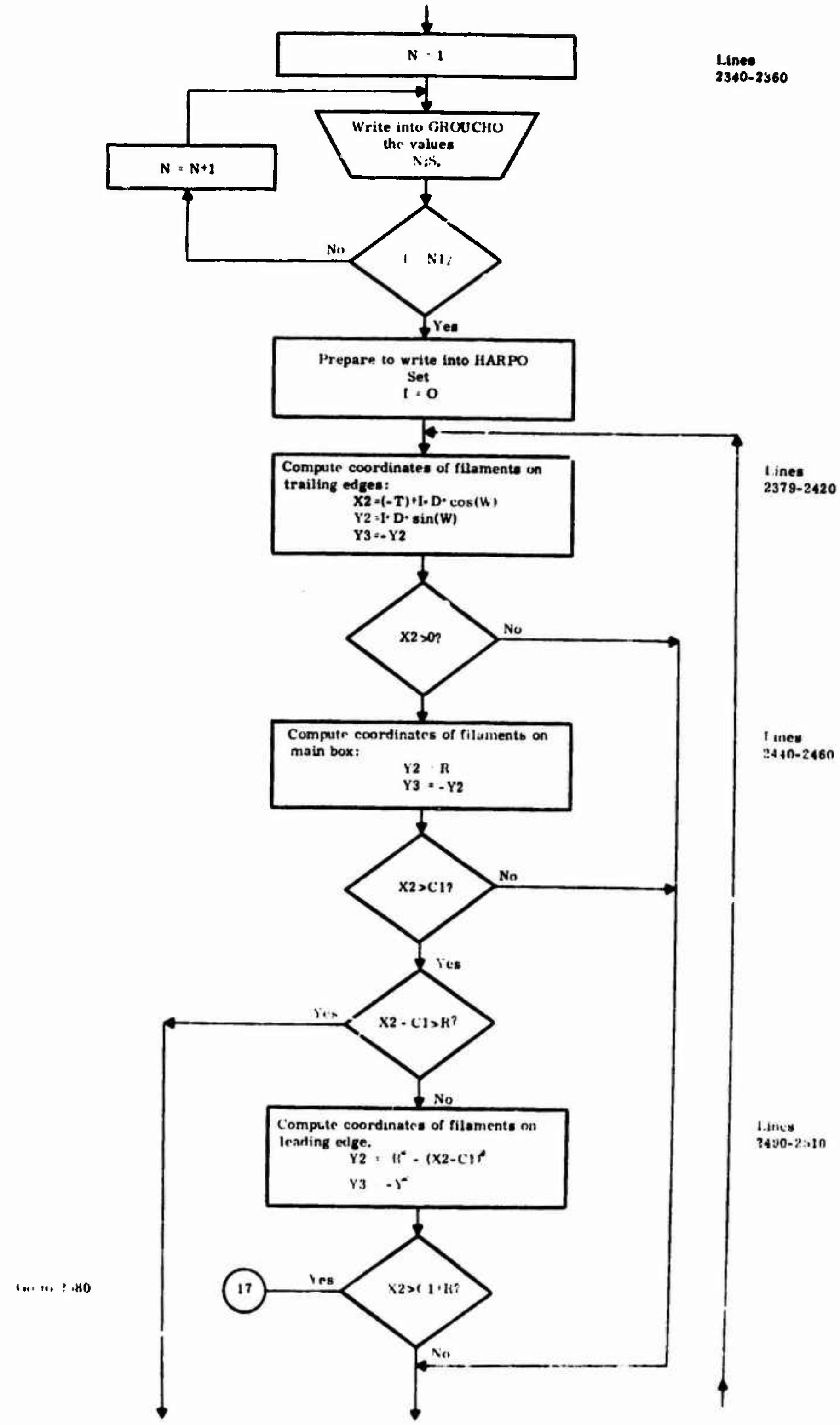


Figure 28. Detailed Flow Diagram (Sheet 12 of 23)

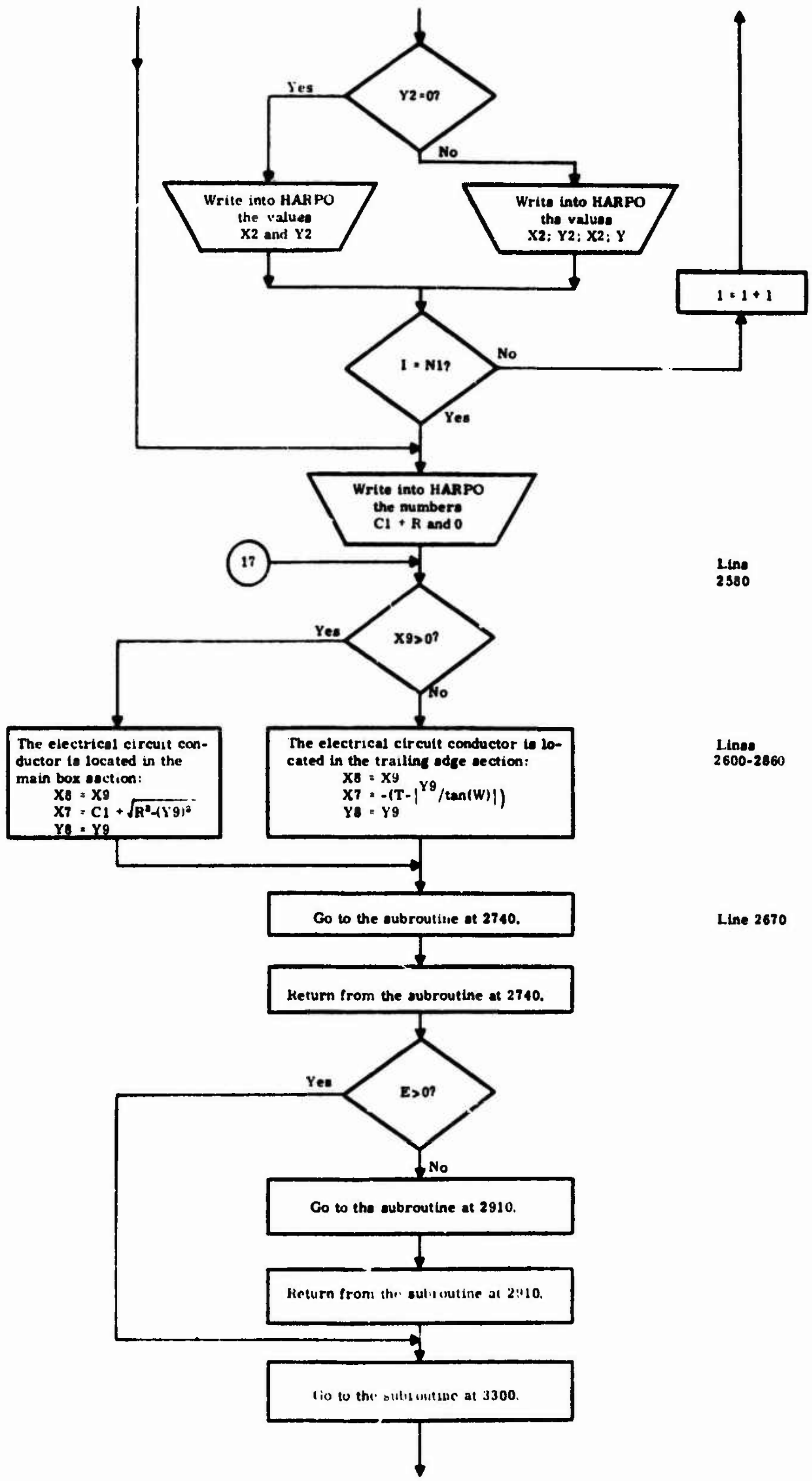


Figure 28. Detailed Flow Diagram (Sheet 13 of 23)

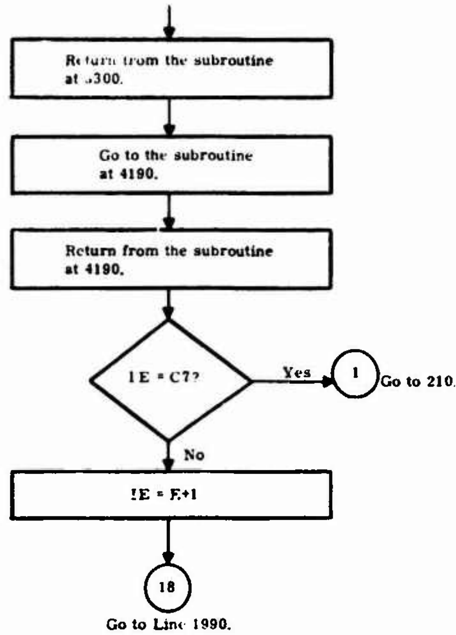


Figure 28. Detailed Flow Diagram (Sheet 14 of 23)

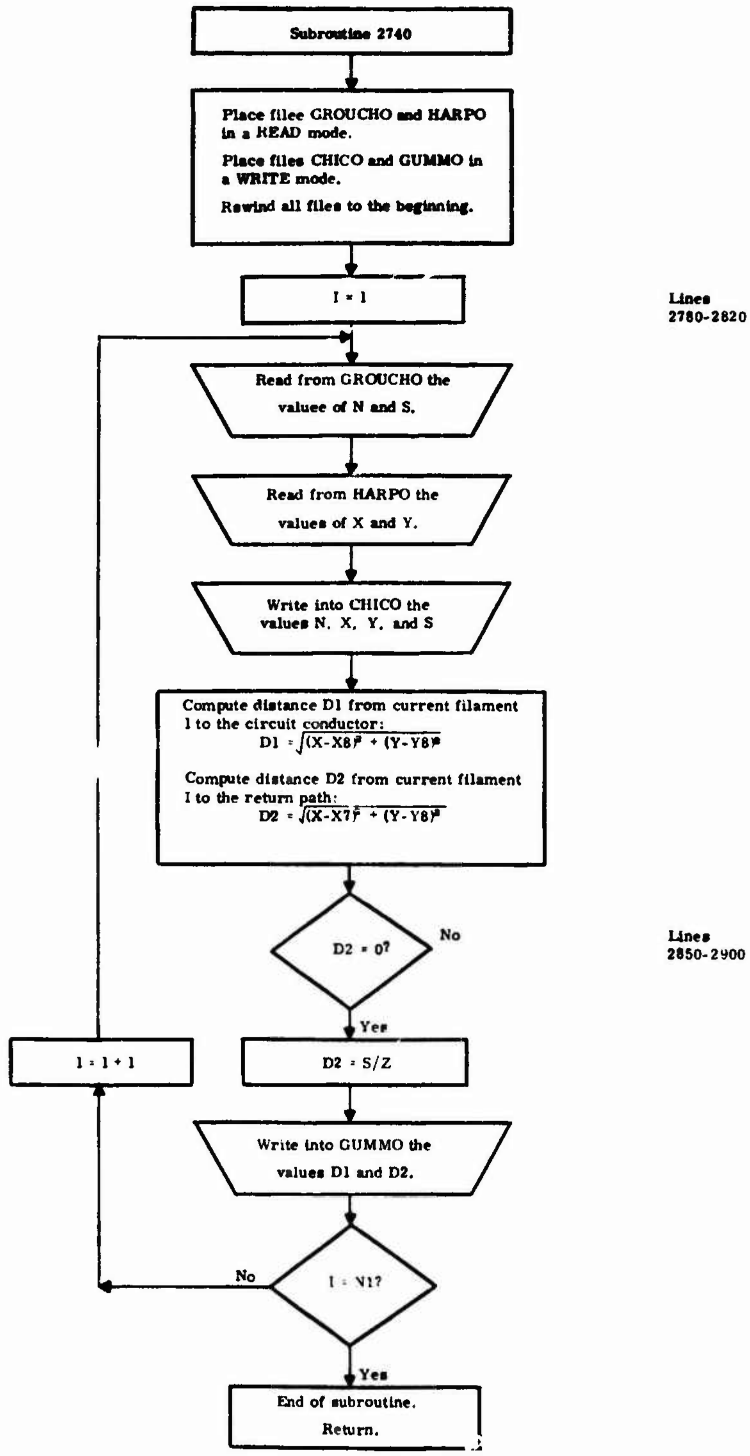


Figure 28. Detailed Flow Diagram (Sheet 15 of 23)

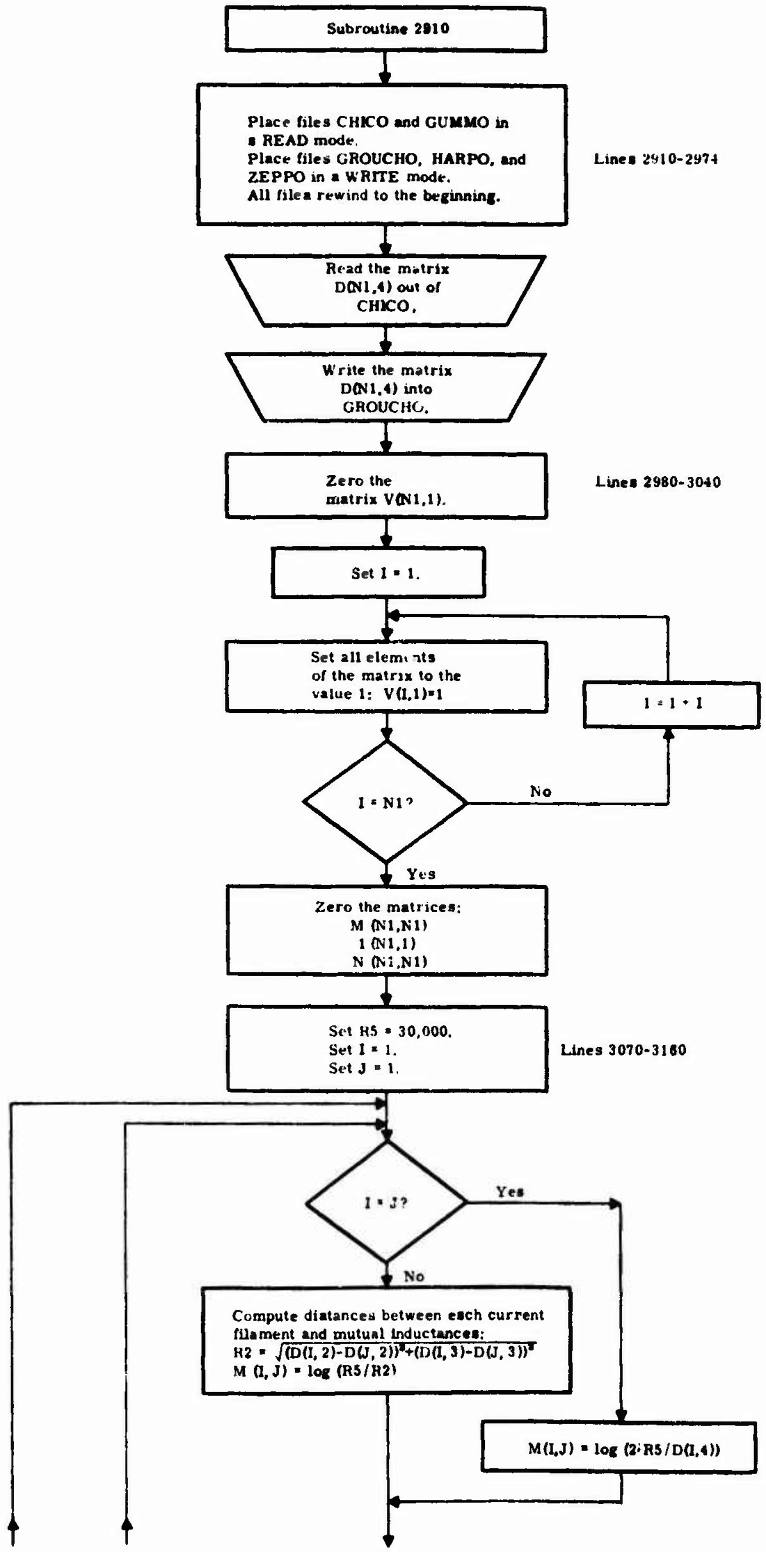
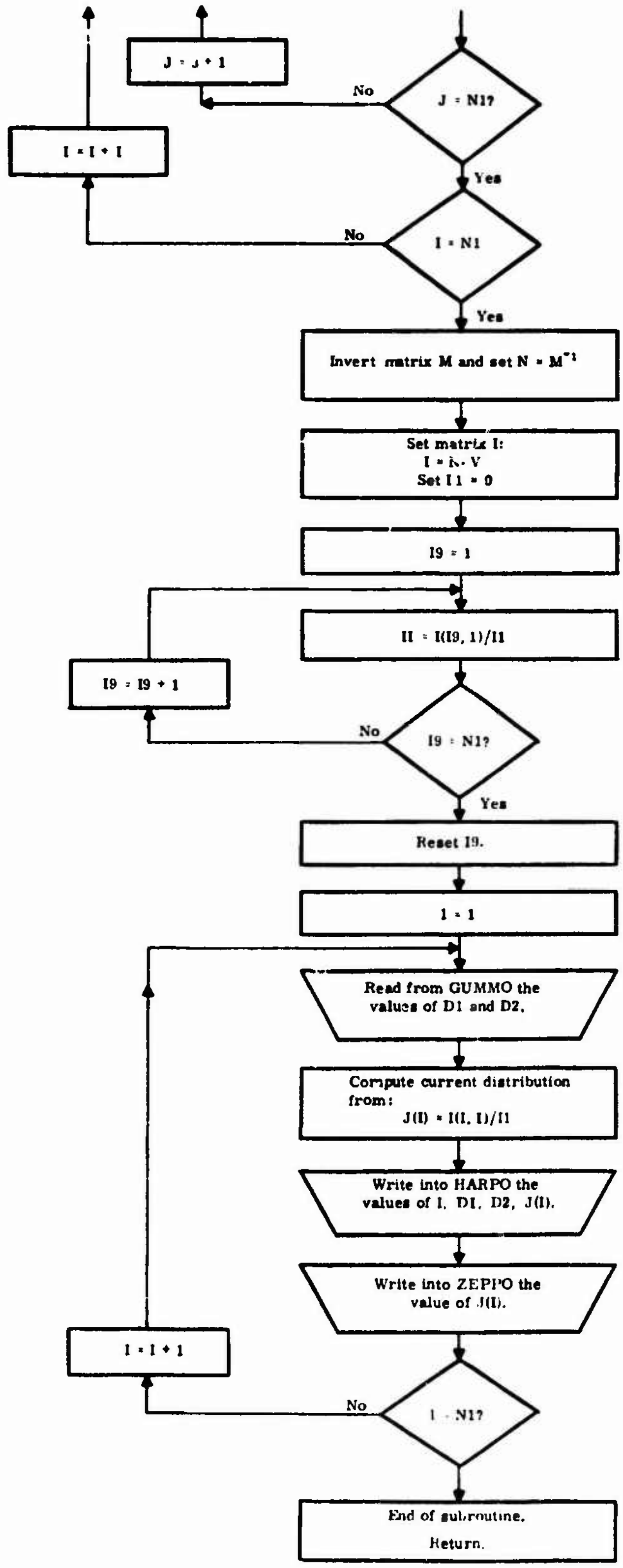


Figure 28. Detailed Flow Diagram (Sheet 16 of 23)



Lines 3170-3220

Lines 3230-3290

Figure 28. Detailed Flow Diagram (Sheet 17 of 23)

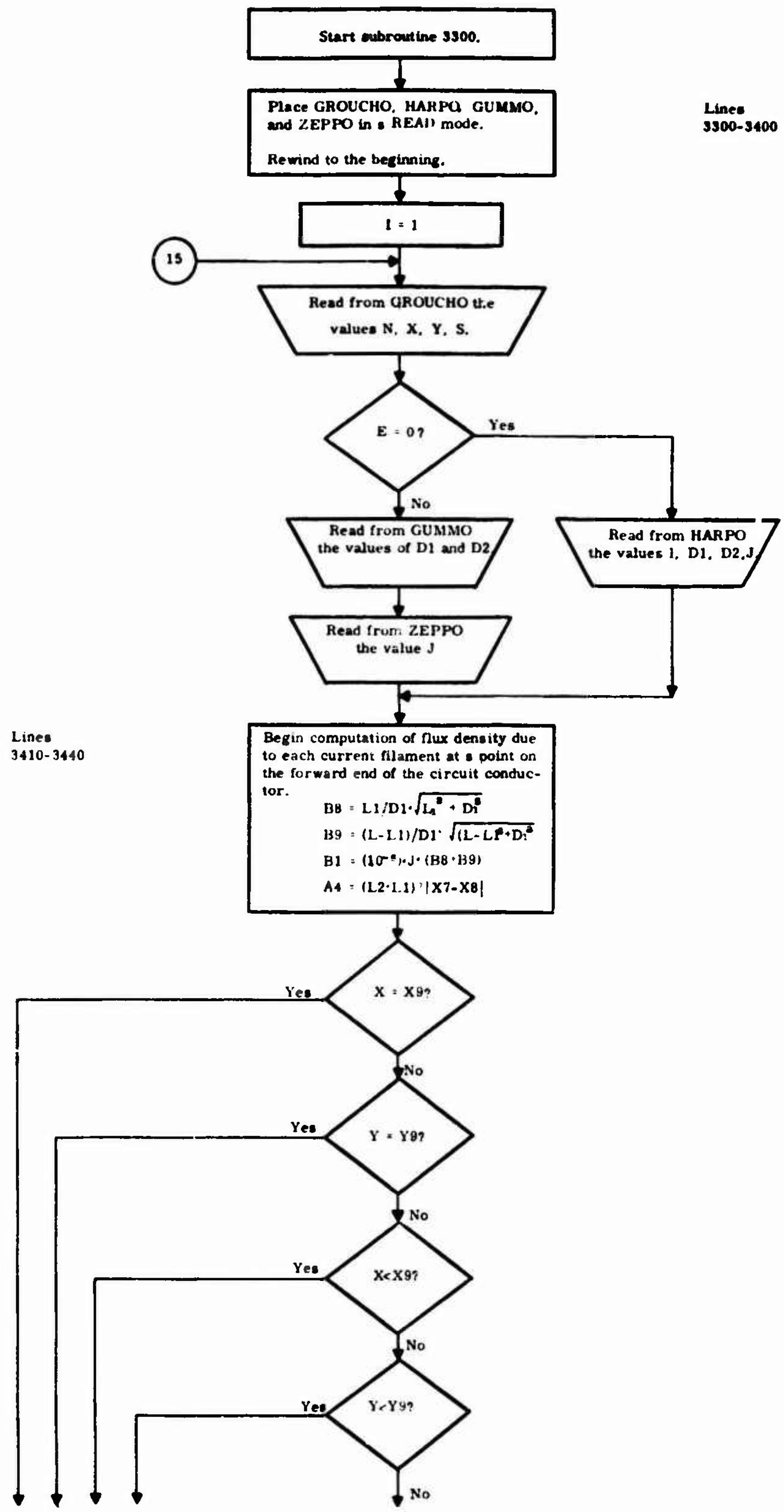


Figure 28. Detailed Flow Diagram (Sheet 18 of 23)

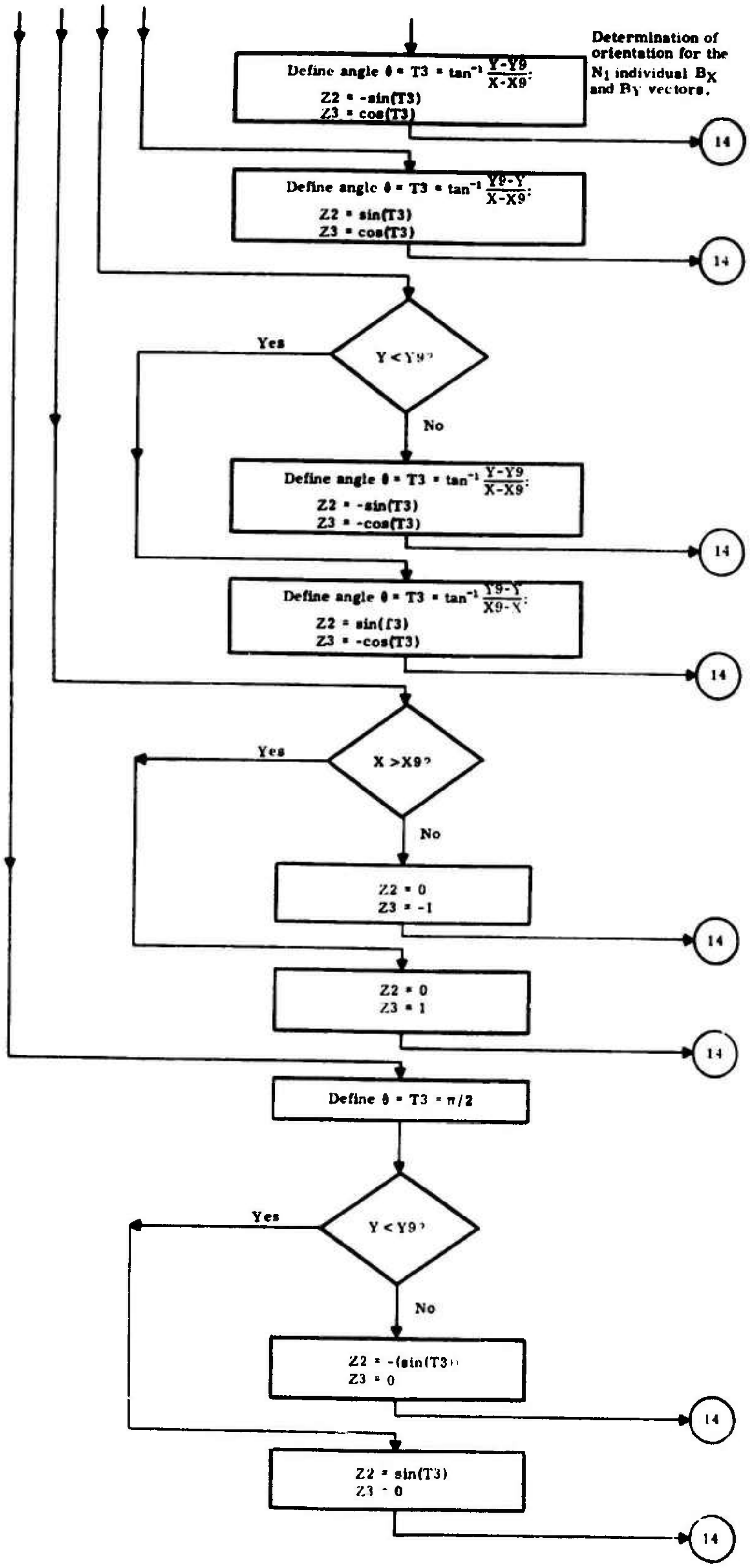


Figure 28. Detailed Flow Diagram (Sheet 19 of 23)

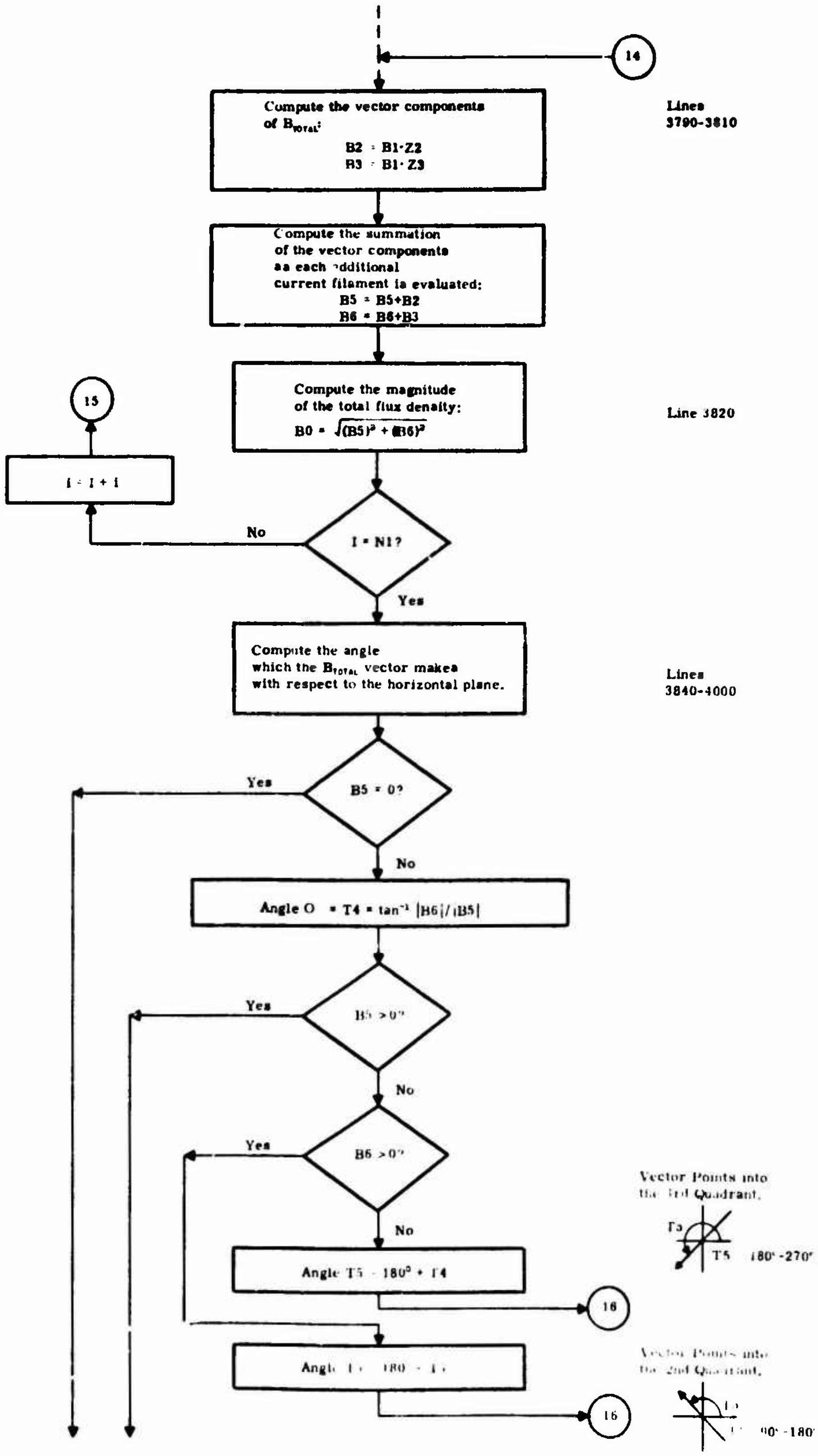


Figure 28. Detailed Flow Diagram (Sheet 20 of 23)

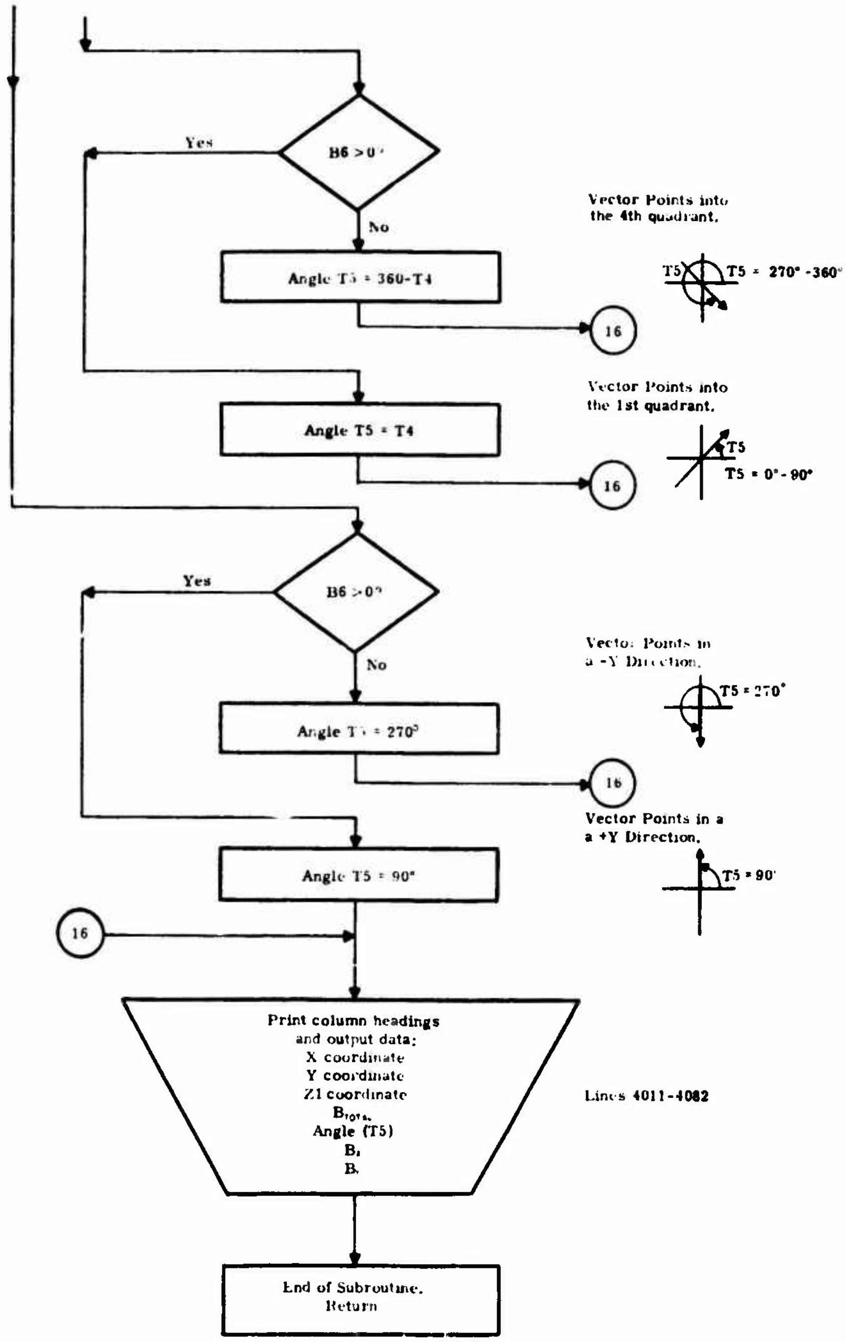


Figure 28. Detailed Flow Diagram (Sheet 21 of 23)

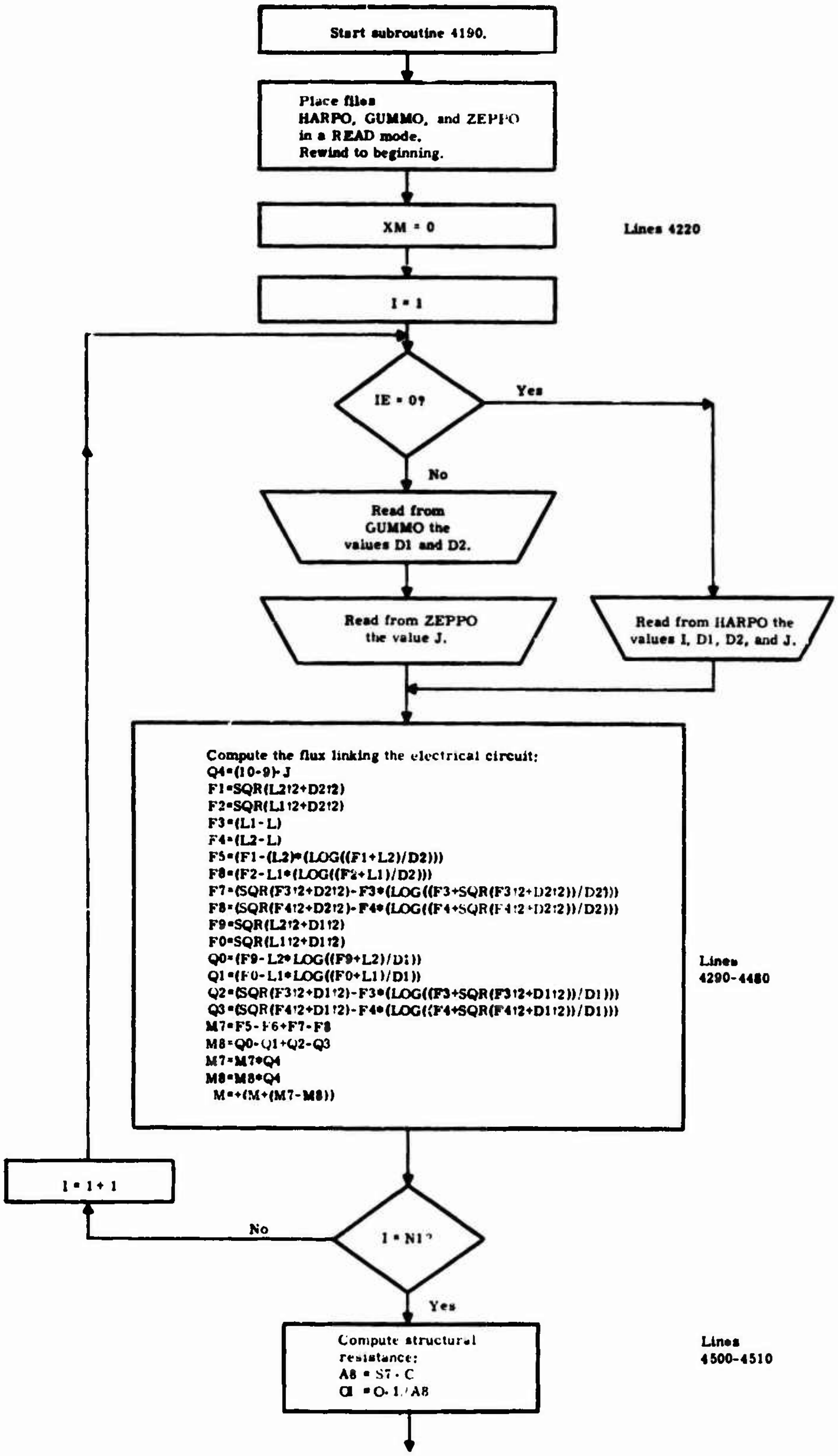


Figure 28. Detailed Flow Diagram (Sheet 22 of 23)

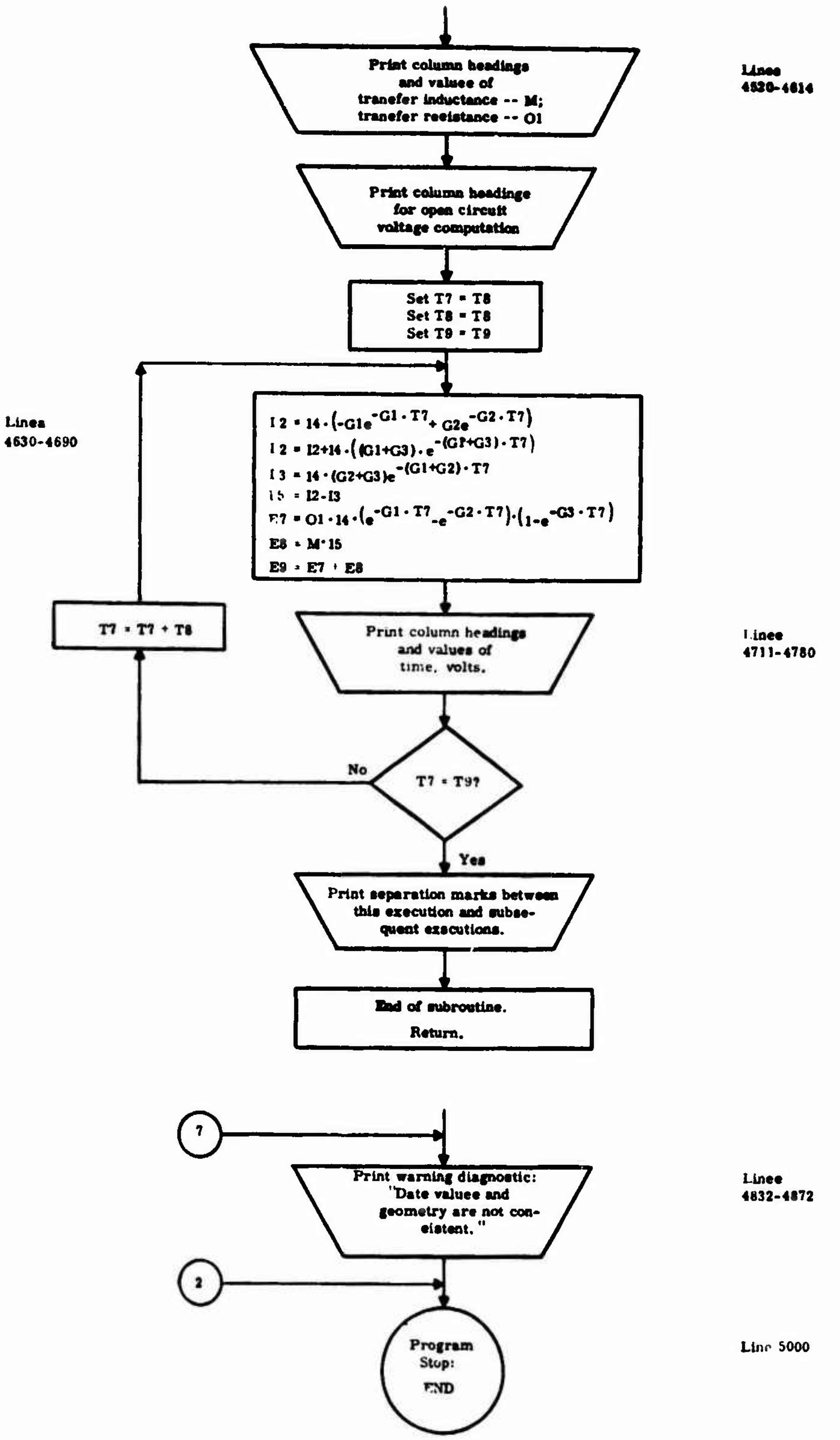


Figure 28. Detailed Flow Diagram (Sheet 23 of 23)

```
1C DIFFUSION------A COMPUTER PROGRAM WHICH CALCULATES THE
2C                 DIFFUSION FIELDS AND THE DIFFUSION COUPLED
3C                 VOLTAGES INTERIOR TO SEVERAL AIRCRAFT
4C                 GEOMETRICAL COMPONENTS.
5C
6C
7C    KEITH J. MAXWELL BLDG 9-209 GENERAL ELECTRIC COMPANY
8C    100 WOODLAWN AVE. PITTSFIELD,MASS. 01201
9C    PHONE (413)-494-3531.
10C
11C
12C DEVELOPED UNDER CONTRACT F33611-74-C-3068 USAF FLIGHT
13C DYNAMICS LABORATORY.
14C
15C
16C THE PROGRAM READS DATA FROM AN EXTERNAL FILE THE NAME
17C OF WHICH HAS BEEN SET TO "MAXWELL".THE INPUT DATA SHOULD
18C BE ARRANGED AS FOLLOWS FOR FUSELAGE GEOMETRIES:
19C
20C    LINE NUMBER  100 A
21C                 110 A1,R1,R2,X1,Y1,C7,X5,D3,D4,D5,Y5,D6,D7,D8
22C                 120 S,L7,O,L3,O3,O4,O5,L4,O6,O7,O8,T8,T9,I4,
23C                     G1,G2,G3,G4
24C                 130 A
25C                 140 ---------SAME AS ABOVE USING 2ND DATA SET------
26C LINE NUMBERS MAY BE ADDED INDEFINITELY UNTIL ALL CASES HAVE
27C BEEN DESCRIBED .
28C
29C
30C DATA ARRANGEMENT FOR WING,HORIZ STAB,AND VERT STAB
31C SHOULD BE AS FOLLOWS:
32C
33C    LINE NUMBER 100 A
34C                110 A2,R,C1,T,S,L7,C7,X5,D3,D4,D5,Y5,D6,D7,D8
35C                120 O,L3,O3,O4,O5,L4,O6,O7,O8,T8,T9,I4,
36C                    G1,G2,G3,G4
37C                130 A
38C                140-------SAME AS ABOVE USING 2ND DATA SET--------
39C ADDITIONAL LINES OF DATA MAY BE USED UNTIL ALL CASES ARE
40C DESCRIBED. GEOMETRIES MAY BE MIXED OR SEPARATED AS DESIRED.
41C
42C
43C A DESCRIPTION OF THE VARIABLES FOLLOWS:
44C
45C  A........THE VALUE OF A ROUTES THE PROGRAM TO THE APPROPRIATE
46C          GEOMETRICAL CONFIGURATION.
47C          A=0-----STOP!
48C          A=1-----FUSELAGE
49C          A=2-----WING
50C          A=3-----HORIZ STAB
```

Figure 29. DIFFUSION Program Listing for the General Electric Time Sharing Computer (Sheet 1 of 17)

```
51C           A=4-----VERT STAB
52C THE VALUES A1,A2,A3,A4 ARE USED AS A CØMPARISØN WITH THE
53C VALUE ØF A TØ INSURE THAT THE INPUT DATA CØRRESPØNDS TØ THE
54C GEØMETRY SPECIFIED.
55C  THE VALUES ØF R1 AND R2 ARE THE RADIUS ØF CURVATURE ØF
56C THE TØP CØRNERS AND THE BØTTØM CØRNERS ØF THE FUSELAGE
57C RESPECTIVELY.
58C X1 AND Y1 ARE THE HEIGHT AND WIDTH ØF THE FUSELAGE.
59C C7 IS USED TØ DECIDE HØW MANY RELØCATIØNS ØF A CIRCUIT
60C CØNDUCTØR ARE TØ BE MADE.
61C X5 AND Y5 ARE THE INITIAL X-Y CØØRDINATES ØF A CIRCUIT
62C CØNDUCTØR. THE CIRCUIT BEGINS AT A DEPTH ØF L3 INSIDE
63C THE FUSELAGE AND EXTENDS TØ THE DISTANCE L4.
64C A SET ØF MØDIFIERS IS PRØVIDED FØR EACH VALUE DESCRIBING
65C THE LØCATIØN ØF THE CIRCUIT. THESE MØDIFIERS CHANGE THE
66C ØRIGINAL PØSITIØN ØF THE CIRCUIT BY A STEP SIZE GIVEN
67C AS :         X-----STEPPED BY AN AMØUNT D3
68C              Y-----STEPPED BY AN AMØUNT D6
69C             L3-----STEPPED BY AN AMØUNT Ø3
70C             L4-----STEPPED BY AN AMØUNT Ø6
71C STEPPING BEGINS AT     E=D4
72C                        E=D7
73C                        E=Ø4
74C                        E=Ø7
75C FØR THE VARIABLES X,Y,L3,L4 RESPECTIVELY
76C STEPPING ØF ANY ØNE VARIABLE TERMINATES WHEN
77C                        E=D5--------X=XMAX
78C                        E=D8--------Y=YMAX
79C                       EE=Ø5--------L3=L3MAX
80C                        E=Ø8--------L4=L4MAX
81C THE PRØGRAM EXECUTES ØVER THE RANGE ØF A DØ LØØP
82C FRØM E=0 TØ E=C7.
83C THE VARIABLE S SPECIFIES THE AVERAGE SKIN THICKNESS.
84C THE VARIABLE Ø SPECIFIES THE RESISTIVITY IN ØHM-CM FØR THE
85C TYPE ØF MATERIAL WHICH CØMPRISES THE SKIN.
86C FØR EACH ITERATIØN A CØMPUTATIØN IS MADE ØF THE
87C FLUX DENSITY THE TRANSFER INDUCTANCE, ANDTHE
88C TRANSFER RESISTANCE.
89C ADDITIØNALLY ,FØR A SPECIFIED LIGHTNING WAVESHAPE
90C A TABULATIØN ØF ØPEN CIRCUIT VØLTAGE VS. TIME IS MADE.
91C FØR A TIME PERIØD T8 TØ T9 IN STEPS ØF T8 (USECS).
92C THE WAVESHAPE IS CHARACTERIZED BY A DØUBLE EXPØNENTIAL ØR A DAMPED
93C SINWAVE MØDIFIED BY THE DIFFUSIØN TIME CØNSTANT.
94C THE CHØICE ØF WAVESHAPE IS MADE BY THE USER.FØR A SINWAVE
95C SET THE VARIABLE G4=1.0.ANY ØTHER VALUE DEFAULTS TØ THE DØUBLE
96C EXPØNENTIAL.BØTH TYPES ØF EQUATIØNS ARE SPECIFIED WHEN THE
97C USER SELECTS VALUES FØR THE VARIABLES "I4","G1","G2",AND "G3".
98C SEE THE USERS MANUAL FØR SUGGESTED VALUES TØ BE USED.
99C FØR WING--HØRIZ--VERT  DATA , (LINES 33-38) ,  R IS THE
100C LEADING EDGE RADIUS,C1 IS THE FWD TØ AFT LENGTH ØF THE
```

Figure 29. DIFFUSION Program Listing for the General Electric Time Sharing Computer (Sheet 2 of 17)

```
101C MAIN BOX (STRAIGHT SECTION),T IS THE FWD TO AFT LENGTH
102C OF THE FLAPS (WING),TAPERED TRAILING EDGE (HORIZ STAB)
103C OR RUDDER (VERT STAB)
110 INTEGER A,A1,A2,C7,D4,D5,D7,D8,E,I,J,K,LM,N,N1,O4,O5,O7,O8
111 REAL A8,B0,B1,B2,B3,B5,B6,B8,B9,C,C1,D,XMATD,D1,D2,D3,D6
112 REAL E2,E6,E7,E8,E9,F0,F1,F2,F3,F4,F5,F6,F7,F8,F9,G1,G2,G3
113 REAL H1,H2,H3,XMATI,I1,I2,I3,I4,I5,XJ,K1,K2,K3,K4,L1,L2,L3,L4,XL
114 REAL XN,XMATM,N7,M8,XMATN,O,O1,O3,O6,P1,P2,Q0,Q1,Q2,Q3,Q4
115 REAL R,R1,R2,R5,S,S7,T,T1,T3,T4,T5,T7,T8,T9,V,XMATV,W
116 REAL X,X1,X2,X5,X7,X8,X9,Y,Y1,Y2,Y3,Y5,Y8,Y9,Z2,Z3
117 90 FORMAT(V)
118 DINENSION FILES(6)
119 FILENAME FILES/"GROUCHO","HARPO","CHICO","GUMMO","ZEPPO","MAXWELL"/
120 DINENSION XMATM(16,16),XMATV(16,1),XMATI(16,1),XNATD(16,4)
121 DIMENSION XMATN(16,16),XMATJ(16)
122  REWIND "GROUCHO"
123 ENDFILE "GROUCHO"
124  REWIND "HARPO"
125 ENDFILE "HARPO"
126  REWIND "CHICO"
127 ENDFILE "CHICO"
128  REWIND "GUMMO"
151 ENDFILE "GUMMO"
160  REWIND "ZEPPO"
161 ENDFILE "ZEPPO"
170  P1=3.1415927
180  P2=6.2831853
190  E2=2.71828
200 N1=16
210 210 READ("MAXWELL",90) A
220 IF(A.EQ.0)GOTO5000
230 GO TO (240,1840,1860,1880,5000),A
240 240 READ("MAXWELL",90)A1,R1,R2,X1,Y1,C7,X5,D3,D4,D5,Y5,D6,D7,D8
250 READ( "MAXWELL",90)S,XL,O,L3,O3,O4,O5,L4,O6,O7,O8,T8,T9,I4,G1,G2,G38
261 PRINT 262
262 262 FORNAT(1H ,17X,"**DIFFUSION--COUPLING--IN--FUSELAGE**")
270 PRINT 272
271 PRINT 272
272 272 FORMAT(1H/)
280 PRINT 272
286 IEUP=C7+1
287 DO 1820 IEDUM=1,IEUP
288 IE=IEDUM-1
290  REWIND "GROUCHO"
291 ENDFILE "GROUCHO"
300  REWIND "HARPO"
301 ENDFILE "HARPO"
310  REWIND "CHICO"
311 ENDFILE "CHICO"
320  REWIND "GUMMO"
```

Figure 29. DIFFUSION Program Listing for the General Electric Time Sharing Computer (Sheet 3 of 17)

```
321 ENDFILE "GUMMO"
330  S7=S
350 IF(IE.EQ.C7)GOTO 1830
360  X9=X5
370  Y9=Y5
380  L1=L3
390  L2=L4
400 IF(D4.LE.IE)GOTO420
410 GOTO440
420 420  X9=X5+(D3*(IE-D4))
430 IF(D5.LT.IE)GOTO570
440 440 IF(D7.LE.IE)GOTO460
450 GOTO480
460 460  Y9=Y5+(D6*(IE-D7))
470 IF(D8.LT.IE)GOTO590
480 480 IF(O4.LE.IE)GOTO500
490 GOTO520
500 500  L1=L3+(O3*(IE-O4))
510 IF(O5.LT.IE)GOTO610
520 520 IF(O7.LE.IE)GOTO540
530 GOTO640
540 540  L2=L4+(O6*(IE-O7))
550 IF(O8.LE.IE)GOTO640
560 GOTO630
570 570  X9=X5+(D3*D5)
580 GOTO440
590 590  Y9=Y5+(D6*D8)
600 GOTO480
610 610  L1= L3+(O3*O5)
620 GOTO520
630 630  L2=L4+(O6*O8)
640 640  H1=X1-R1
650  K1=Y1-R1
660  H2=R1
670  K2=Y1-R1
680  H3=R2
690  K3=R2
700  H4=X1-R2
710  K4=R2
720 IF(A.EQ.A1)GOTO740
730 GOTO4820
740 740 CONTINUE
741C REM
742 REWIND "GROUCHO"
743 ENDFILE "GROUCHO"
744 REWIND "HARPO"
745 ENDFILE "HARPO"
750 DO770I=1,N1
760 WRITE ("GROUCHO",90)I,S
770 770 CONTINUE
```

Figure 29. DIFFUSION Program Listing for the General Electric Time Sharing Computer (Sheet 4 of 17)

```
780  C=2*(Y1-(R1+R2))+2*(X1-2*R2)+P1*(R1+R2)
790  D=C/N1
800  X=0
809 IUP=N1+1
810 DO850IDUM=1,IUP
811 I=IDUM-1
820  Y=(Y1-R1)-D*I
830 IF(Y.LE.R2)GOTO860
840 WRITE ("HARPO",90)X,Y
850 850 CONTINUE
860 860 IF(X.EQ.X1)GOTO930
870  X=X1
879 IUP=N1+1
880 DO920IDUM=1,IUP
881 I=IDUM-1
890  Y=R2+D*I
900 IF(Y.GE.Y1-R1)GOTO930
910 WRITE ("HARPO",90)X,Y
920 920 CONTINUE
930 930  Y=0
940 DO980J=1,N1
950  X2=R2+D*J
960 IF(X2.GE.X1-R2)GOTO990
970 WRITE ("HARPO",90)X2,Y
980 980 CONTINUE
990 990 IF(Y.EQ.Y1)GOTO1060
1000  Y=Y1
1009 IUP=N1+1
1010 DO1050IDUM=1,IUP
1011 I=IDUM-1
1020  X=(X1-R1)-D*I
1030 IF(X.LE.R1)GOTO1060
1040 WRITE ("HARPO",90)X,Y
1050 1050 CONTINUE
1060 1060 KUP=N1+1
1061  DO1120KDUM=1,KUP
1062 K=KDUM-1
1070  T1=K*D/R2
1080  X2=H3-R2*COS(T1)
1090  Y2=K3-R2*SIN(T1)
1100 IF(X2.GE.R2)GOTO1130
1110 WRITE ("HARPO",90)X2,Y2
1120 1120 CONTINUE
1130 1130 CONTINUE
1131C REM RESET
1139 KUP=N1+1
1140 DO1200KDUM=1,KUP
1141 K=KDUM-1
1150  T1=K*D/R2
1160  X2=H4+R2*SIN(T1)
```

Figure 29. DIFFUSION Program Listing for the General Electric Time Sharing Computer (Sheet 5 of 17)

```
1170  Y2=K4-R2*CØS(T1)
1180 IF(Y2.GE.R2)GØTØ1210
1190 WRITE ("HARPØ",90)X2,Y2
1200 1200 CØNTINUE
1210 1210 CØNTINUE
1211C REM RESET
1219 LUP=N1+1
1220 DØ1280LDUM=1,LUP
1221 L=LDUM-1
1230  T1=L*D/R1
1240  X2=H1+R1*CØS(T1)
1250  Y2=K1+R1*SIN(T1)
1260 IF(X2.LE.H1)GØTØ1290
1270 WRITE ("HARPØ",90)X2,Y2
1280 1280 CØNTINUE
1290 1290 CØNTINUE
1291C REM RESET
1299 MUP=N1+1
1300 DØ1360MDUM=1,MUP
1301 M=MDUM-1
1310  T1=M*D/R1
1320  X2=H2-R1*SIN(T1)
1330  Y2=K2+R1*CØS(T1)
1340 IF(Y2.LE.K2)GØTØ1370
1350 WRITE ("HARPØ",90)X2,Y2
1360 1360 CØNTINUE
1370 1370 CØNTINUE
1371C REM RESET
1380 IF(X9.LE.R2)GØTØ1420
1390 IF(X9.LE.R1)GØTØ1430
1400 IF(X9.GE.H4)GØTØ1540
1410 IF(X9.GE.H1)GØTØ1550
1420 1420 IF(Y9.LE.R2)GØTØ1450
1430 1430 IF(Y9.GE.K1)GØTØ1500
1440 GØTØ1680
1450 1450 IF(X9.EQ.Y9)GØTØ1690
1460  X8=X9
1470  Y8=Y9
1480  X7=(R2-SQRT(R2↑2-((R2-Y9)↑2)))
1490 GØTØ1760
1500 1500  X8=X9
1510  X7=R1-(SQRT((R1)↑2-((R1-Y1+Y9)↑2)))
1520  Y8=Y9
1530 GØTØ1760
1540 1540 IF(Y9.LE.R2)GØTØ1600
1550 1550 IF(Y9.GE.K1)GØTØ1640
1560  X8=X9
1570  X7=X1
1580  Y8=Y9
1590 GØTØ1760
```

Figure 29. DIFFUSION Program Listing for the General Electric Time Sharing Computer (Sheet 6 of 17)

```
1600 1600  X8=X9
1610  X7=X1+(SQRT(R2↑2-(R2-Y9)↑2))-R2
1620  Y8=Y9
1630 GØTØ1760
1640 1640  X8=X9
1650  X7=X1+(SQRT((R1)↑2-(R1-Y1+Y9)↑2))-R1
1660  Y8=Y9
1670 GØTØ1760
1680 1680 IF(X9.GE.X1/2)GØTØ1730
1690 1690  X8=X9
1700  X7=0
1710  Y8=Y9
1720 GØTØ1760
1730 1730  X8=X9
1740  X7=X1
1750  Y8=Y9
1760 1760 CØNTINUE
1761C REM
1770 ASSIGN 1778 TØ SW2900
1772 GØ TØ 2740
1778 1778 CØNTINUE
1780 IF(IE.GT.0)GØTØ1800
1790 ASSIGN 1798 TØ SW3290
1792 GØ TØ 2910
1798 1798 CØNTINUE
1800 1800 ASSIGN 1808 TØ SW4180
1802 GØ TØ 3300
1808 1808 CØNTINUE
1810 ASSIGN 1818 TØ SW4810
1812 GØ TØ 4190
1818 1818 CØNTINUE
1820 1820 CØNTINUE
1830 1830 GØTØ210
1841 1840 PRINT 1842
1842 1842 FØRMAT(1H ,20X,"**DIFFUSIØN--CØUPLING--IN--WINGØ*")
1850 GØTØ1890
1861 1860 PRINT 1862
1862 1862 FØRMAT(1H ,10X,"**DIFFUSIØN--CØUPLING--IN--HØRIZØNTAL
1863&--STABILIZER**")
1870 GØTØ1890
1881 1880 PRINT 1882
1882 1882 FØRMAT(1H ,11X,"DIFFUSIØN--CØUPLING--IN--VERTICAL
1883&--STABILIZER**")
1890 1890 PRINT 272
1900 PRINT 272
1910 READ( "MAXWELL",90)A2,R,C1,T,S,XL,C7,X5,D3,D4,D5,Y5,D6,D7,D8
1920 READ( "MAXWELL",90)Ø,L3,Ø3,Ø4,Ø5,L4,Ø6,Ø7,Ø8,T8,T9,I4,G1,G2,G3,G4
1930  REWIND "GRØUCHØ"
1931 ENDFILE "GRØUCHØ"
1940  REWIND "HARPØ"
```

Figure 29. DIFFUSION Program Listing for the General Electric Time Sharing Computer (Sheet 7 of 17)

```
1941 ENDFILE "HARPO"
1950  REWIND "CHICO"
1951 ENDFILE "CHICO"
1960  REWIND "GUMMO"
1961 ENDFILE "GUMMO"
1970  S7=S
1979 IEUP=C7+1
1980 DO 2720 IEDUM=1,IEUP
1981 IE=IEDUM-1
1990 IF(IE.EQ.C7)GOTO 2730
2000  X9=X5
2010  Y9=Y5
2020  L1=L3
2030  L2=L4
2040 IF(D4.LE.IE)GOTO2060
2050 GOTO2080
2060 2060  X9=X5+(D3*(IE-D4))
2070 IF(D5.LT.IE)GOTO2210
2080 2080 IF(D7.LE.IE)GOTO2100
2090 GOTO2120
2100 2100  Y9=Y5+(D6*(IE-D7))
2110 IF(D8.LT.IE)GOTO2230
2120 2120 IF(O4.LE.IE)GOTO2140
2130 GOTO2160
2140 2140  L1=L3+(O3*(IE-O4))
2150 IF(O5.LT.IE)GOTO2250
2160 2160 IF(O7.LE.IE)GOTO2180
2170 GOTO2280
2180 2180  L2=L4+(O6*(IE-O7))
2190 IF(O8.LE.IE)GOTO2280
2200 GOTO2270
2210 2210  X9=X5+(D3*D5)
2220 GOTO2080
2230 2230  Y9=Y5+(D6*D8)
2240 GOTO2120
2250 2250  L1= L3+(O3*O5)
2260 GOTO2160
2270 2270  L2=L4+(O6*O8)
2280 2280 IF(A.EQ.A2)GOTO2300
2290 GOTO4820
2300 2300  W=ATAN(R/T)
2310  REWIND "GROUCHO"
2311 ENDFILE "GROUCHO"
2320  C=(P1*R)+2*C1+2*(SQRT(R↑2+T↑2))
2330  D=C/N1
2340 DO2360N=1,N1
2350 WRITE ("GROUCHO",90)N,S
2360 2360 CONTINUE
2370  REWIND "HARPO"
2371 ENDFILE "HARPO"
```

Figure 29. DIFFUSION Program Listing for the General Electric Time Sharing Computer (Sheet 8 of 17)

```
2379 IUP=N1+1
2380 DØ2570IDUM=1,IUP
2381 I=IDUM-1
2390  X2=(-T)+(I*D*CØS(W))
2400  Y2=I*D*SIN(W)
2410  Y3=-Y2
2420 IF(X2.GT.0)GØTØ2440
2430 GØTØ2530
2440 2440  Y2=R
2450  Y3=-Y2
2460 IF(X2.GT.C1)GØTØ2480
2470 GØTØ2530
2480 2480 IF(X2-C1.GT.R)GØTØ2580
2490  Y2=SQRT((R↑2)-(X2-C1)↑2)
2500  Y3=-Y2
2510 IF(X2.GT.C1+R)GØTØ2590
2520 GØTØ2530
2530 2530 IF(Y2.EQ.0)GØTØ2560
2538 WRITE("HARPØ",90) X2,Y2
2540 WRITE ("HARPØ",90)X2,Y3
2550 GØTØ2570
2560 2560 WRITE ("HARPØ",90)X2,Y2
2570 2570 CØNTINUE
2580 2580 WRITE ("HARPØ",90)C1+R,0
2590 2590 IF(X9.GT.0)GØTØ2640
2600  X8=X9
2610 X7=-(T-(ABS(Y9/(TAN(W)))))
2620  Y8=Y9
2630 GØTØ2670
2640 2640  X8=X9
2650  X7=C1+SQRT(R↑2-(Y9)↑2)
2660  Y8=Y9
2670 2670 ASSIGN 2678 TØ SW2900
2672 GØ TØ 2740
2678 2678 CØNTINUE
2680 IF(IE.GT.0)GØTØ2700
2690 ASSIGN 2698 TØ SW 3290
2692 GØ TØ 2910
2698 2698 CØNTINUE
2700 2700 ASSIGN 2708 TØ SW4180
2702 GØ TØ 3300
2708 2708 CØNTINUE
2710 ASSIGN 2718 TØ SW4810
2712 GØ TØ 4190
2718 2718 CØNTINUE
2720 2720 CØNTINUE
2730 2730 GØTØ210
2740 2740 REWIND "GRØUCHØ"
2750 REWIND "HARPØ"
2760  REWIND "CHICØ"
```

Figure 29. DIFFUSION Program Listing for the General Electric Time Sharing Computer (Sheet 9 of 17)

```
2761 ENDFILE "CHICØ"
2770  REWIND "GUMMØ"
2771 ENDFILE "GUMMØ"
2780 DØ2890I=1,N1
2790 READ( "GRØUCHØ",90)N,S
2800 READ( "HARPØ",90)X,Y
2820 WRITE ("CHICØ",90)N,X,Y,S
2830  D1=SQRT((X-X8)↑2+(Y-Y8)↑2)
2840  D2=SQRT((X-X7)↑2+(Y-Y8)↑2)
2850 IF(D2.EQ.0)GØTØ2870
2860 GØTØ2880
2870 2870  D2=S/2
2880 2880 WRITE ("GUMMØ",90)D1,D2
2890 2890 CØNTINUE
2900 GØ TØ SW2900
2910 2910 REWIND "CHICØ"
2920 REWIND "GUMMØ"
2930  REWIND "GRØUCHØ"
2931 ENDFILE "GRØUCHØ"
2940  REWIND "HARPØ"
2941 ENDFILE "HARPØ"
2950  REWIND "ZEPPØ"
2951 ENDFILE "ZEPPØ"
2964 READ("CHICØ",90) ((XMATD(IRØW,ICØL),ICØL=1,4),IRØW=1,N1)
2970 DØ 2978 IRØW=1,N1
2974 WRITE("GRØUCHØ",90) (XMATD(IRØW,ICØL),ICØL=1,4)
2978 2978 CØNTINUE
2980 CALL MATZER(XMATV,N1,1)
2990 DØ3010I=1,N1
3000  XMATV(I,1)=1
3010 3010 CØNTINUE
3020 CALL MATZER(XMATM,N1,N1)
3030 CALL MATZER(XMATI,N1,1)
3040 CALL MATZER(XMATN,N1,N1)
3050  P2=6.28318
3060  E2=2.71828
3070  R5=30000
3076 3076 FØRMAT((4(1H ,G13.5)/))
3079 3079 FØRMAT((5(1H ,G13.5)/))
3080 DØ3160I=1,N1
3090 DØ3150J=1,N1
3100 IF(I.EQ.J)GØTØ3140
3110  R3=SQRT((XMATD(I,2)-XMATD(J,2))↑2+(XMATD(I,3)-XMATD(J,3))↑2)
3120  XMATM(I,J)=ALØG(R5/R3)
3130 GØTØ3150
3140 3140  XMATM(I,J)=ALØG(R5*2/XMATD(I,4))
3150 3150 CØNTINUE
3160 3160 CØNTINUE
3170 CALL MATINV(XMATM,XMATN,N1,N1)
3172 DØ 3174 I9=1,N1
```

Figure 29. DIFFUSION Program Listing for the General Electric Time Sharing Computer (Sheet 10 of 17)

```
3173 XMATI(I9,1)=0
3174 3174 CONTINUE
3175 DO 3179 I9=1,N1
3176 DO 3178 J9=1,N1
3177 XMATI(I9,1)=XMATI(I9,1)+XMATN(I9,J9)
3178 3178 CONTINUE
3179 3179 CONTINUE
3189 3189 FORMAT(G13.5/)
3190  I1=0
3200 DO3220I=1,N1
3210  I1=XMATI(I,1)+I1
3220 3220 CONTINUE
3230 DO3280I=1,N1
3240 READ( "GUMMO",90)D1,D2
3250  XMATJ(I)=XMATI(I,1)/I1
3260 WRITE ("HARPO",90)I,D1,D2,XMATJ(I)
3270 WRITE ("ZEPPO",90)XMATJ(I)
3280 3280 CONTINUE
3290 GO TO SW3290
3300 3300 REWIND "CHICO"
3310 REWIND "HARPO"
3320 REWIND "GUMMO"
3330 REWIND "ZEPPO"
3340 DO3830I=1,N1
3350 READ( "CHICO",90)XDUMN,X,Y,S
3360 IF(IE.EQ.0)GOTO3400
3370 READ( "GUMMO",90)D1,D2
3380 READ( "ZEPPO",90)XJ
3390 GOTO3410
3400 3400 READ( "HARPO",90)I1,D1,D2,XJ
3410 3410  B8=(L1)/(D1*(SQRT((L1↑2)+(D1↑2))))
3420  B9=((XL-L1))/(D1*(SQRT((XL-L1)↑2+(D1↑2))))
3430  B1=((1E-5)*XJ)*(B8+B9)
3435 A4=(L2-L1)*(ABS(X7-X8))
3440 IF(X.EQ.X9)GOTO3640
3450 IF(Y.EQ.Y9)GOTO3720
3460 IF(X.LT.X9)GOTO3560
3470 IF(Y.LT.Y9)GOTO3520
3480  T3=ATAN((Y-Y9)/(X-X9))
3490  Z2=-(SIN(T3))
3500  Z3=COS(T3)
3510 GOTO3780
3520 3520  T3=ATAN((Y9-Y)/(X-X9))
3530  Z2=SIN(T3)
3540  Z3=COS(T3)
3550 GOTO3780
3560 3560 IF(Y.LT.Y9)GOTO3610
3570  T3=ATAN((Y-Y9)/(X9-X))
3580  Z2=-(SIN(T3))
3590  Z3=-(COS(T3))
```

Figure 29. DIFFUSION Program Listing for the General Electric Time Sharing Computer (Sheet 11 of 17)

```
3600 GOTO3780
3610 3610  T3=ATAN((Y9-Y)/(X9-X))
3620  Z2=SIN(T3)
3630  Z3=-(COS(T3))
3635 GO TO 3780
3640 3640  T3=PI/2
3650 IF(Y.LT.Y9)GOTO3690
3660  Z2=-(SIN(T3))
3670  Z3=0
3680 GOTO3780
3690 3690  Z2=SIN(T3)
3700  Z3=0
3710 GOTO3780
3720 3720 IF(X.GT.X9)GOTO3760
3730  Z2=0
3740  Z3=-1
3750 GOTO3780
3760 3760  Z2=0
3770  Z3=1
3780 3780  B2=B1*Z2
3790  B3=B1*Z3
3800  B5=B5+B2
3810  B6=B6+B3
3820  B0=SQRT((B5↑2)+(B6↑2))
3830 3830 CONTINUE
3840 IF(B5.EQ.0)GOTO3970
3850  T4=ATAN(ABS(B6)/ABS(B5))
3860 IF(B5.GT.0)GOTO3920
3870 IF(B6.GT.0)GOTO3900
3880  T5=180+(T4*57.2958)
3890 GOTO4010
3900 3900  T5=180-(T4*57.2958)
3910 GOTO4010
3920 3920 IF(B6.GT.0)GOTO3950
3930  T5=360-(T4*57.2958)
3940 GOTO4010
3950 3950  T5=T4*57.2958
3960 GOTO4010
3970 3970 IF(B6.GT.0)GOTO4000
3980  T5=270
3990 GOTO4010
4000 4000  T5=90
4011 4010 PRINT 4012
4012 4012 FORMAT(1H ,"MAGNETIC.......................FIELD
4013&..................COMPUTATION")
4030 PRINT 272
4031 PRINT 4032,X9
4032 4032 FORMAT(1H ,"X-COORDINATE=",G13.6)
4033 PRINT 4034,Y9
4034 4034 FORMAT(1H ,"Y-COORDINATE=",G13.6)
```

Figure 29. DIFFUSION Program Listing for the General Electric Time Sharing Computer (Sheet 12 of 17)

```
4036 PRINT 4037,L1
4037 4037 FØRMAT(1H ,"Z1-CØØRDINATE=",G13.5)
4038 PRINT 4039,L2
4039 4039 FØRMAT(1H ,"Z2-CØØRDINATE=",G13.5)
4041 PRINT 4042
4042 4042 FØRMAT(1H ,1X,"LØØP AREA         B-X                B-Y
4043&        B-TØTAL          ANGLE")
4061 PRINT 4062
4062 4062 FØRMAT(1H ,39X,"(WEBERS/METER↑2)    (DEGREES)")
4070 PRINT 272
4081 PRINT 4082,A4,B5,B6,B0,T5
4082 4082 FØRMAT(1H ,5(1H ,G13.6))
4120  B0=0
4130  B1=0
4140  B2=0
4150  B3=0
4160  B5=0
4170  B6=0
4180 GØ TØ SW4180
4190 4190 REWIND "HARPØ"
4200 REWIND "GUMMØ"
4210 REWIND "ZEPPØ"
4220  XM=0
4230 DØ4490I=1,N1
4240 IF(IE.EQ.0)GØTØ4280
4250 READ( "GUMMØ",90)D1,D2
4260 READ( "ZEPPØ",90)XJ
4270 GØTØ4290
4280 4280 READ( "HARPØ",90)II,D1,D2,XJ
4290 4290  Q4=(1E-9)*XJ
4300  F1=SQRT(L2↑2+D2↑2)
4310  F2=SQRT(L1↑2+D2↑2)
4320  F3=(L1-XL)
4330  F4=(L2-XL)
4340  F5=(F1-(L2)*(ALØG((F1+L2)/D2)))
4350  F6=(F2-L1*(ALØG((F2+L1)/D2)))
4360  F7=(SQRT(F3↑2+D2↑2)-F3*(ALØG((F3+SQRT(F3↑2+D2↑2))/D2)))
4370  F8=(SQRT(F4↑2+D2↑2)-F4*(ALØG((F4+SQRT(F4↑2+D2↑2))/D2)))
4380  F9=SQRT(L2↑2+D1↑2)
4390  F0=SQRT(L1↑2+D1↑2)
4400  Q0=(F9-L2*ALØG((F9+L2)/D1))
4410  Q1=(F0-L1*ALØG((F0+L1)/D1))
4420  Q2=(SQRT(F3↑2+D1↑2)-F3*(ALØG((F3+SQRT(F3↑2+D1↑2))/D1)))
4430  Q3=(SQRT(F4↑2+D1↑2)-F4*(ALØG((F4+SQRT(F4↑2+D1↑2))/D1)))
4440  M7=F5-F6+F7-F8
4450  M8=Q0-Q1+Q2-Q3
4460  M7=M7*Q4
4470  M8=M8*Q4
4480  XM=(XM+(M7-M8))
4490 4490 CONTINUE
```

Figure 29. DIFFUSION Program Listing for the General Electric Time Sharing Computer (Sheet 13 of 17)

```
4500  A8=S7*C
4510  Ø1=Ø*(XL/A8)
4520 PRINT 272
4530 PRINT 272
4541 PRINT 4542
4542 4542 FØRMAT(1H ,2X,"TRANSFER+-+-+-+-+-+-+-+-+-+-+-+-FUNCTIØN
4543&+-+-+-+-+-+-+-+-+-+-CØMPUTATIØN")
4560 PRINT 272
4571 PRINT 4572
4572 4572 FØRMAT(1H ,8X,"TRANSFER INDUCTANCE
4573&                    TRANSFER RESISTANCE")
4581 PRINT 4582
4582 4582 FØRMAT(1H ,13X,"(HENRIES)                              (ØHMS)")
4591 PRINT 4592,XM,Ø1
4592 4592 FØRMAT(1H ,12X,G13.6,22X,G13.6)
4601 PRINT 4602
4602 4602 FØRMAT(1H ,"ØPEN       CIRCUIT       VØLTAGE")
4610 PRINT 272
4613 PRINT 4614
4614 4614 FØRMAT(1H ,"TIME                        VØLTS")
4619 T7=0
4620 DØ 4720 IDUMMY=1,999
4621 T7=T7+T8
4622 IF (T7.GT.T9) GØ TØ 4721
4623 IF (G4.EQ.1.0) GØ TØ 4692
4630  I2=I4*((-G1*EXP(-G1*T7))+G2*EXP(-G2*T7))
4640  I2=I2+I4*((G1+G3)*EXP((-G1-G3)*T7))
4650  I3=I4*(G2+G3)*EXP((-G2-G3)*T7)
4660  I5=I2-I3
4670  E7=Ø1*I4*(EXP(-G1*T7)-EXP(-G2*T7))*(1-EXP(-G3*T7))
4680  E8=XM*I5
4690  E9=E7-E8
4691 GØ TØ 4711
4692 4692 AMP=(I4*SIN(2.*PI*G1*T7))*EXP(-G2*T7)*(1-EXP(-G3*T7))
4693 DAMPDT=I4*(2.*PI*G1*CØS(2.*PI*G1*T7))
4694 DAMPDT=DAMPDT*(EXP(-G2*T7)-EXP((-G2-G3)*T7))
4695 DAMP=I4*(SIN(2.*PI*G1*T7))
4696 DAMPT=DAMP*((G2+G3)*EXP((-G2-G3)*T7)-G1*EXP(-G1*T7))
4697 DAMPDT=DAMPDT+DAMPT
4698 E7=Ø1*AMP
4699 E8=XM*DAMPDT
4700 E9=E7-E8
4711 4711 PRINT 4712,T7,E9
4712 4712 FØRMAT(1H ,G13.6,3H   ,G13.6)
4720 4720 CØNTINUE
4721 4721 CØNTINUE
4730 PRINT 272
4741 PRINT 4742
4742 4742 FØRMAT(1H ,75(1H=))
4760 PRINT 272
```

Figure 29. DIFFUSION Program Listing for the General Electric Time Sharing Computer (Sheet 14 of 17)

```
4770 PRINT 272
4780 PRINT 272
4790  XM=0
4800  01=0
4810 GO TO SW4810
4820 4820 PRINT 272
4831 PRINT 4832
4832 4832 FORMAT(1H ,"DATA READ STATEMENT DOES NOT CONTAIN")
4841 PRINT 4842
4842 4842 FORMAT(1H ,"VALUES WHICH CORRESPOND TO THIS")
4851 PRINT 4852
4852 4852 FORMAT(1H ,"GEOMETRY.CHECK ALL DATA STATEMENTS")
4861 PRINT 4862
4862 4862 FORMAT(1H ,"TO BE SURE THAT THEY ARE CONSISTENT")
4871 PRINT 4872
4872 4872 FORMAT(1H ,"WITH THE GEOMETRY YOU ARE EVALUATING.")
5000 5000 STOP;END
5010 SUBROUTINE MATZER(XM,IROW,ICOL)
5020 DIMENSION XM(IROW,ICOL)
5040 DO 3200 I9=1,IROW
5050 DO 3190 J9=1,ICOL
5060 XM(I9,J9)=0
5070 3190 CONTINUE
5080 3200 CONTINUE
5090 RETURN
5100 END
5110 SUBROUTINE MATINV(A,B,IROW,ICOL)
5120 DIMENSION A(IROW,ICOL),B(IROW,ICOL),C(1,1)
5140 DO 3540 I9=1 ,IROW
5150 DO 3538 J9=1 , ICOL
5160 B(I9,J9)=A(I9,J9)
5170  3538 CONTINUE
5180 3540 CONTINUE
5190 CALL MATRIX(6,B,A,C,IROW,ICOL,IROW,ICOL,ICOL)
5200 RETURN
5210 END
6000 SUBROUTINE MATRIX(IOP,A,B,C,I,J,K,L,M)
6010 REAL A,B,C,TEMP
6020 DIMENSION A(I,J),B(I,J),C(I,J)
6030 DIMENSION LABEL(16)
6040 GO TO (101,102,103,104,200,300,400), IOP
6050 101 ASSIGN 111 TO IP; GO TO 100
6060 102 ASSIGN 112 TO IP; GO TO 100
6070 103 ASSIGN 113 TO IP; GO TO 100
6080 104 ASSIGN 114 TO IP
6090 100 DO 120 I1=1,K
6100 DO 120 I2=1,L
6110 GO TO IP,(111,112,113,114)
6120 111 C(I1,I2)=A(I1,I2)+B(I1,I2); GO TO 120
6130 112 C(I1,I2)=A(I1,I2)-B(I1,I2); GO TO 120
```

Figure 29. DIFFUSION Program Listing for the General Electric Time Sharing Computer (Sheet 15 of 17)

```
DIFUSION    02/06/75

6140 113 C(I1,I2)=A(I1,I2)*B(I1,I2)$ GO TO 120
6150 114 C(I1,I2)=A(I1,I2)/B(I1,I2)
6160 120 CONTINUE
6170 GO TO 500
6180 200 DO 210 I1=1,K
6190 DO 210 I2=1,L
6200 TEMP=0.
6210 DO 205 I3=1,M
6220 205 TEMP=TEMP+A(I1,I3)*B(I3,I2)
6230 210 C(I1,I2)=TEMP
6240 GO TO 500
6250 300 NR=K$ NC=K
6260 DO 21 J1=1,NR
6270 21 LABEL(J1)=J1
6280 DO 291 J1=1,NR
6290 TMP1=0.
6300 DO 121 J2=J1,NR
6310 TMP2=CABS(A(J2,J1))
6320 IF(TMP2-TMP1) 121,121,1210
6330 1210 TMP1=TMP2
6340 IBIG=J2
6350 121 CONTINUE
6360 IF(IBIG.EQ.J1) GO TO 201
6370 DO 141 J2=1,NC
6380 TEMP=A(J1,J2)
6390 A(J1,J2)=A(IBIG,J2)
6400 141 A(IBIG,J2)=TEMP
6410 I=LABEL(J1)
6420 LABEL(J1)=LABEL(IBIG)
6430 LABEL(IBIG)=I
6440 201 TEMP=A(J1,J1)
6450 A(J1,J1)=1.0
6460 DO 221 J2=1,NC
6470 221 A(J1,J2)=A(J1,J2)/TEMP
6480 DO 281 J2=1,NR
6490 IF(J2.EQ.J1) GO TO 281
6500 TEMP=A(J2,J1)
6510 A(J2,J1)=0.
6520 DO 241 J3=1,NC
6530 241 A(J2,J3)=A(J2,J3)-TEMP*A(J1,J3)
6540 281 CONTINUE
6550 291 CONTINUE
6560 301 N1=NR-1
6570 DO 391 J1=1,N1
6580 DO 321 J2=J1,NR
6590 IF(LABEL(J2).NE.J1) GO TO 321
6600 IF(J2.EQ.J1) GO TO 391
6610 GO TO 341
6620 321 CONTINUE
6630 341 DO 361 J3=1,NR
```

Figure 29. DIFFUSION Program Listing for the General Electric Time Sharing Computer (Sheet 16 of 17)

```
DIFUSION    02/06/75

6640 TEMP=A(J3,J1)
6650 A(J3,J1)=A(J3,J2)
6660 361 A(J3,J2)=TEMP
6670 LABEL(J2)=LABEL(J1)
6680 391 CONTINUE
6690 GO TO 500
6700 400 DO 410 I1=1,K
6710 DO 410 I2=1,L
6720 410 C(I2,I1)=A(I1,I2)
6730 500 RETURN;END
```

Figure 29. DIFFUSION Program Listing for the General Electric Time Sharing Computer (Sheet 17 of 17)

etries. Once the particular geometry being evaluated has been defined, the program defines a horizontal plane containing both the electrical circuit conductor and a structural return path, and stores the coordinates of this plane for utilization later in the program. This is done in lines 1380-1760 or 2590-2708. At this point in the program, the information that has been generated is:

- Location of the enclosed electrical circuit conductor
- Coordinates of all current carrying filaments representing the geometrical structure
- Loop or surface for which flux linkage is to be computed

The program now branches to a subroutine (lines 2680-2900) which determines the distance, D1, from each of the skin current filaments to the enclosed conductor, and additionally computes the distance, D2, from each of the skin current filaments to the return skin conductor. These values are retained and stored in one of the files for later use.

The next computation to be performed is the current distribution at each filament. The computer program partitions the input lightning current and defines for each of the current filaments a fractional portion of the total current. The portion of the lightning current assigned to each current filament depends on the geometry and the location of the current filament in that geometry; this operation is executed in lines 2910-3290. At this point in the program execution, the computer program has defined the location of each current carrying filament, the current distribution in each of these current filaments, and the location of the enclosed electrical circuit conductor. It has also defined a loop through which flux linkages are to be computed.

The program now branches to the subroutine in which flux density equations are used to compute the flux density, its vector components, and its orientation at a given point inside the geometry of interest. The point selected for this computation depends upon the user's selection of the enclosed

circuit conductor initial location, Z1. The computational operations to obtain the flux density are performed in lines 3300-4180.

The computer program then branches to the subroutine that computes the transfer inductance, M, and the transfer resistance, R. In execution of this subroutine a computation is made of the open circuit voltage versus time, utilizing the transfer functions. Computation of the transfer inductance is performed by reading in the previously computed and stored values of D1 and D2 as well as the value of the current distribution for each of the current filaments. These values are inserted into the flux equation for each filament out to a distance D1, which is subtracted from the flux that is computed for the skin current filament out to the distance D2. This difference in flux is the net flux linking the defined plane. The transfer inductance is the summation of all of these individual fluxes divided by the total lightning current that flows through the complete structure. These operations are performed in lines 4190-4490.

The transfer resistance is computed using the equation

$$R = \rho L/A \tag{63}$$

where:

ρ = resistivity of the skin material (ohm-cm)
A = cross-sectional area of the geometry skin (m^2)
L = total length of the geometry being evaluated (meters)

This is obtained from lines 4500-4592. After computing the transfer inductance, M, and the transfer resistance, R, the program computes the induced voltage in the specified electrical circuit, utilizing a lightning waveform described by the user's data inputs. Since naturally occurring lightning strokes vary greatly in waveform, the user may select the waveform for which protection is to be designed (one for either a damped oscillatory or a double exponential waveform may be used). The waveform of the portion of lightning current appearing at the inside surface of the skin, and thus in the skin current filaments described by this program, is not the original lightning current waveform. Instead, it is modified by a diffusion time constant in the manner described in Reference 5. This is accomplished directly in the induced voltage equation. The resulting open circuit voltages are then computed and tabulated as a function of time. The user has control over the time duration and increments over which this computation is executed. These operations are performed in lines 4601-4810.

After completing this computation, the program loops back and determines if modifications to the previous data set have been requested. If modifications are to be made, program execution repeats, using this new data set. A new set of transfer functions is then computed along with the corresponding open circuit voltages.

If no modifications to the data have been requested, the program then determines if another geometry has been selected for evaluation. If such a

geometry is to be evaluated, the constants of that geometry, the initial location of a circuit conductor inside that geometry, and the modifiers which will be used to reposition or relocate that electrical circuit conductor are read and the program operates as before. After all modifications in all geometrical models have been completed, the program reaches an end.

The output data returned from the program are the coordinates of the circuit conductor for the execution in progress, flux density, flux density orientation and vector components, transfer inductance, transfer resistance, and a table of lightning induced voltage versus time in the open circuits of interest.

VALIDATION OF DIFFUSION

The criteria used to evaluate the validity of the computer program were (1) to determine if it returned the same answer that could be manually computed for a textbook calculable geometry, and (2) to compare computer generated values to those of aircraft on which experimental measurements are available from which empirically derived transfer functions were available. Two illustrative cases were selected and are presented.

CASE 1

The object is to compute the mutual inductance between a single current filament and a loop with a configuration as shown in Figure 30.

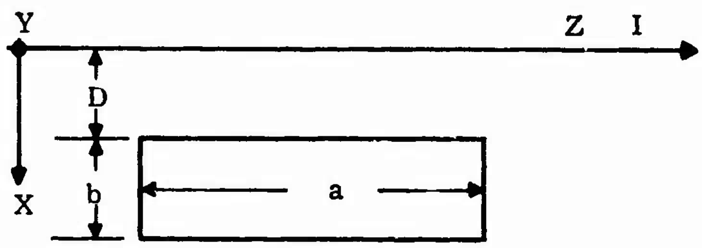


Figure 30. Single Current Carrying Filament and Circuit Loop

The expression which determines the flux linking the loop is obtained for this case from

$$\psi_{12} = \frac{\mu_0 I_1}{2\pi} a \int_{D}^{b+D} \frac{dx}{x} \tag{64}$$

or

$$\psi_{12} = \frac{\mu_0 I_1 a}{2\pi} \ln \frac{b + D}{D} \tag{65}$$

The mutual inductance is obtained by dividing by I; thus,

$$L_{12} = \frac{\psi_{12}}{I} = \frac{\mu_0 a}{2\pi} \ln \frac{b + D}{a} \tag{66}$$

Values were selected for this geometry as follows:

$$a = 50 \text{ cm}; \quad b = 217 \text{ cm}; \quad D = 150 \text{ cm}$$

and L_{12} was computed to be 8.9×10^{-8} henries.

The computer generated value for this case (7.2×10^{-8} henries) is presented in Figure 31.

CASE 2

The object is to compute the flux linking an electrical circuit that is centered in the cylindrical fuselage as shown in Figure 32. Because of the symmetry, the total flux linking this plane should be equal to zero. The computer generated results are shown in Figure 33.

It is evident that in the limit, as the modeled geometry is simplified, the analytical expressions evaluated by the computer program DIFFUSION reduce to easily computed, classical formulas.

MAGNETIC........................FIELD..................COMPUTATION

X-COORDINATE= 43
Y-COORDINATE= 0
Z1-COORDINATE= 315
Z2-COORDINATE= 365

LOOP AREA	B-X	B-Y	B-TOTAL (WEBERS/METER↑2)	ANGLE (DEGREES)
1.085	0	-1.21853E-7	1.21853E-7	270

TRANSFER+-+-+-+-+-+-+-+-+-+-FUNCTION+-+-+-+- -+-+-+-+-+COMPUTATION

TRANSFER INDUCTANCE (HENRIES)	TRANSFER RESISTANCE (OHMS)
7.28114E-8	7.92885E-6

OPEN CIRCUIT VOLTAGE

TIME	VOLTS
0.000001	-1.44704
0.000002	-2.23788
0.000003	-2.57989
0.000004	-2.62368
0.000005	-2.47749
0.000006	-2.21823
0.000007	-1.8996
0.000008	-1.55837
0.000009	-1.21399
0.00001	-0.896995

==

Figure 31. Diffusion Calculated Values of Transfer Functions and Open Circuit Induced Voltage in Single Geometry of Figure 30

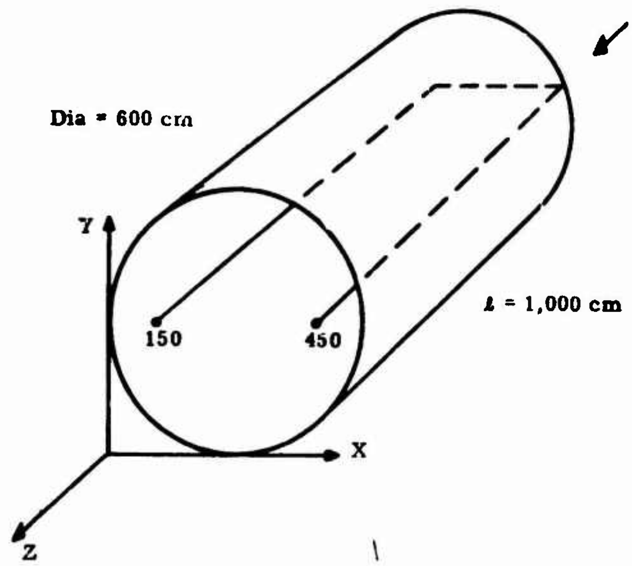


Figure 32. Electrical Circuit Loop Inside a Cylindrical Fuselage

```
   MAGNETIC.....................FIELD.................COMPUTATION

X-COORDINATE= 450
Y-COORDINATE= 300
Z1-COORDINATE= 100
Z2-COORDINATE= 500
 LOOP AREA          B-X                B-Y          B-TOTAL           ANGLE
                                                (WEBERS/METER*2)   (DEGREES)

 120000          4.16334E-17       4.22756E-9       4.22756E-9        90.


  TRANSFER+-+-+-+-+-+-+-+-+-+-+-FUNCTION+-+-+-+-+-+-+-+-+-+-+-+-COMPUTATION

        TRANSFER INDUCTANCE                     TRANSFER RESISTANCE
            (HENRIES)                                 (OHMS)
               0                             4.70458E-6
OPEN        CIRCUIT         VOLTAGE

   TIME             VOLTS
 0.000001        5.11307E-5
 0.000002        1.73543E-4
 0.000003        3.31811E-4
 0.000004        5.01998E-4
 0.000005        6.68474E-4
 0.000006        8.21549E-4
 0.000007        9.55726E-4
 0.000008        1.06841E-3
 0.000009        1.15896E-3
 0.00001         1.22802E-3

=========================================================================
```

Figure 33. Diffusion Coupling in Fuselage

Section 3

APERTURE FIELDS

APERTURE THEORY

EQUIVALENT MAGNETIC DIPOLES

If a magnetic field exists tangentially to a surface in which an aperture exists, the fields induced on the other side of that aperture may be treated as those induced by a dipole of appropriate strength lying in the plane of that aperture (Figure 34). A mathematically tractable aperture is an ellipse of

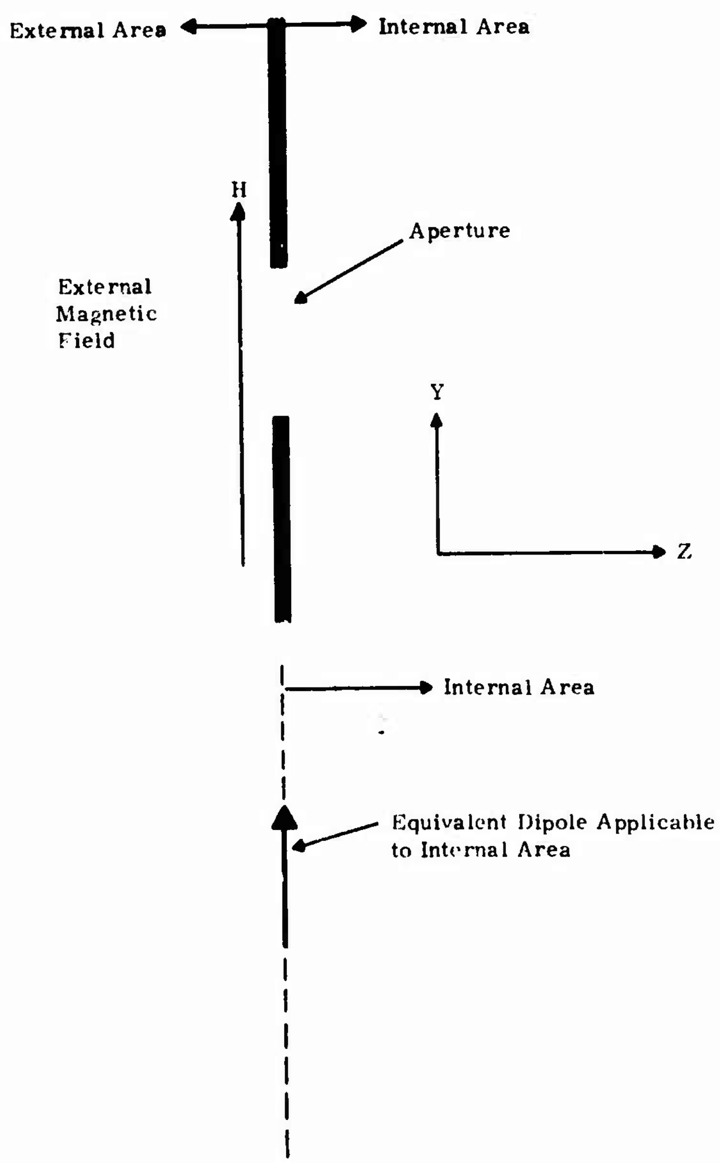


Figure 34. Equivalent Dipole Presented by Aperture

major and minor dimensions, ℓ_1 and ℓ_2. Figure 35 shows such an aperture located in the XY plane. The coordinate structure shown in Figure 35 is referred to in the remainder of this report.

Let H at the aperture be, in vector notation:

$$\overline{H} = H_x + H_y + H_z \qquad (67)$$

where:

$H_x = \alpha_{11}\overline{H}\,(ext)$
$H_y = \alpha_{22}\overline{H}\,(ext)$
$H_z = \alpha_{33}\overline{H}\,(ext)$

$\overline{H}$ is the field strengths that would exist at the aperture if the aperture were not present (Ref. 10).

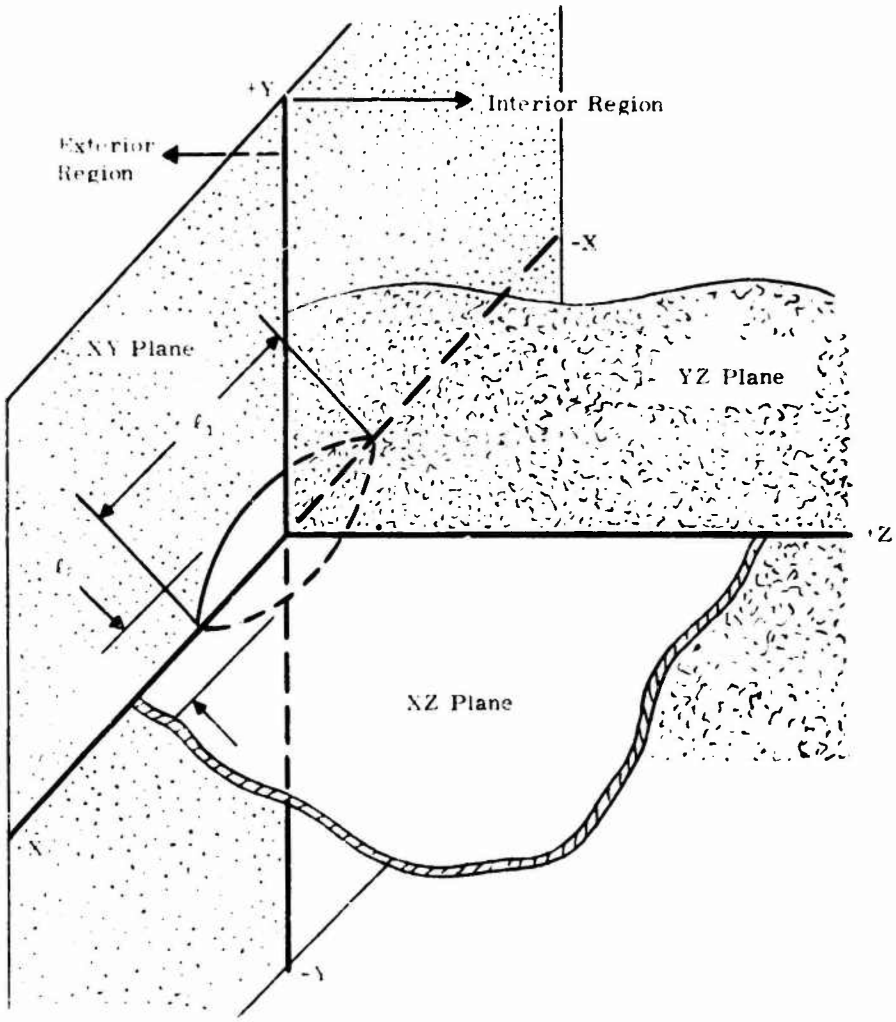


Figure 35. Elliptical Aperture in XY Plane

The α_{11}, α_{22}, and α_{33} are factors related to the shape of the aperture. For the elliptical aperture shown in Figure 35:

$$\alpha_{11} = -\frac{2\pi}{3} \frac{(\ell_{1/2})^3 e^2}{K(e^2) - E(e^2)} \tag{68}$$

$$\alpha_{22} = -\frac{2\pi}{3} \frac{(\ell_{1/2})^3 e^2 (1 - e^2)}{E(e^2) - (1 - e^2) K(e^2)} \tag{69}$$

$$\alpha_{33} = -\frac{2\pi}{3} \frac{(\ell_{1/2})^3 (1 - e^2)}{E(e^2)} \tag{70}$$

where:

$$e^2 = 1 - \left(\frac{\ell_2}{\ell_1}\right)^2$$

and $K(e^2)$ and $E(e^2)$ are elliptic integrals of the first and second kinds, respectively.

At the surface of a conductor the Z component of $\overline{H}$(ext), H_z(ext) must vanish if either the conductance is high enough or the frequency of concern is high enough so the skin depth is small compared to the thickness of the conductor. For the cases of present interest component H_z will frequently be zero, by virtue of the geometry of the current flow producing magnetic field $\overline{H(ext)}$. Likewise, for the cases of present interest, the frequencies at which a magnetic field may penetrate in a Z direction are low enough that they are not of concern. Accordingly, assume that $H_z(ext) = 0$. Under these conditions the equivalent dipoles are:

- Equivalent dipole lying along the X axis:

$$M_x = (H\ell)_x = \alpha_{11} H_x(ext) \tag{71}$$

- Equivalent dipole lying along the Y axis:

$$M_y = (H\ell)_y = \alpha_{22} H_y(ext) \tag{72}$$

For the case in which the major axis of the elliptical aperture is oriented along the X axis (as shown in Figure 35), the values of α_{11}, α_{22}, and α_{33} are given on Figure 36. If the major axis is oriented along the Y axis, the same curve is applicable if the designations of α_{11} and α_{22} are reversed.

FIRST ORDER DIPOLE APPROXIMATION TO INTERNAL MAGNETIC FIELD

Considering a magnetic dipole of strength $H_y\ell$ located along the Y axis, the coordinate geometry would be as shown in Figure 37. At point P the magnetic potential, M, is:

$$M = K_2 \left[\frac{1}{r_1} - \frac{1}{r_2}\right] \tag{73}$$

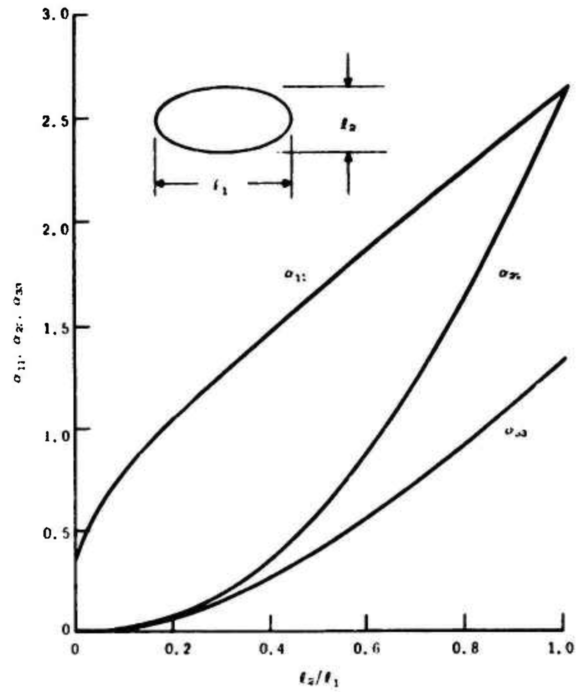
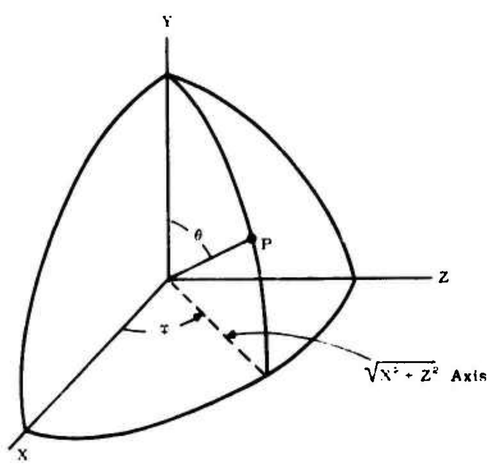
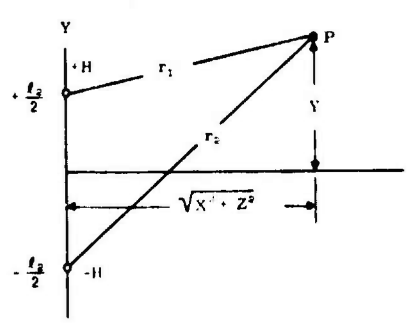


Figure 36. Shape Factor for Elliptical Apertures



a) Coordinate Geometry



b) Details of Dipole Located at Origin

Figure 37. Magnetic Dipole Oriented Along Y Axis

where:

$$K_2 = \frac{H}{4\pi}$$

(For purposes of clarity, constant K_1 is reserved for a later formulation with the dipole located along the X axis.)

Dipole theory generally assumes that point P is sufficiently far from the origin that r_1 and r_2 may be approximated (Figure 38) as:

$$r_1 = r - \frac{\ell_2}{2} \cos\theta \qquad (74)$$

$$r_2 = r + \frac{\ell_2}{2} \cos\theta \qquad (75)$$

Under these circumstances:

$$M = K_2 \left[\frac{1}{r - \frac{\ell_2}{2} \cos\theta} - \frac{1}{r + \frac{\ell_2}{2} \cos\theta} \right] \qquad (76)$$

$$M = K_2 \left[\frac{r + \frac{\ell_2}{2} \cos\theta - r + \frac{\ell_2}{2} \cos\theta}{r^2 - \left(\frac{\ell_2}{2} \cos\theta \right)^2} \right] \qquad (77)$$

If $\ell_2/2 \ll r$, then:

$$M = \frac{K_2 \ell_2 \cos\theta}{r^2} \qquad (78)$$

or:

$$M = \frac{(H\ell_2) \cos\theta}{4\pi r^2} \qquad (79)$$

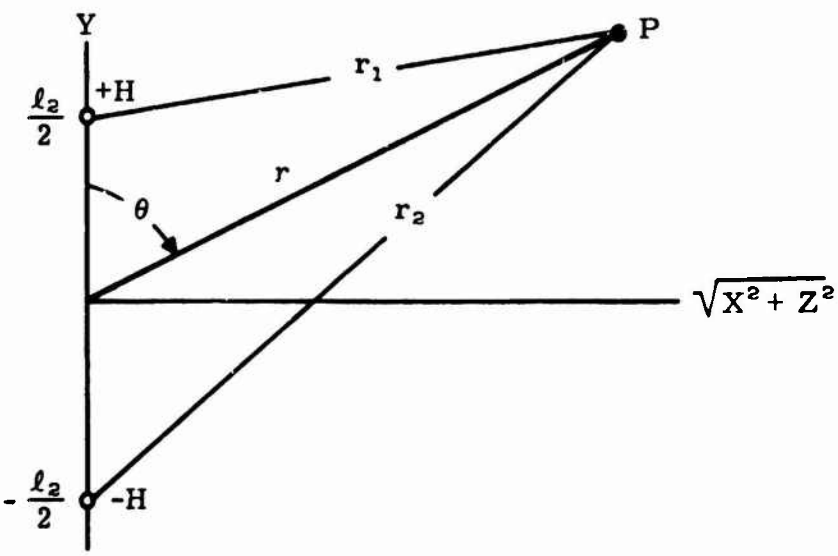


Figure 38. Approximations Used in Elementary Dipole Analysis

HIGHER ORDER APPROXIMATION TO INTERNAL MAGNETIC FIELD

The above formulation is valid when the point at which the field is to be calculated is at a distance, from the aperture, that is large compared to the dimensions of the aperture. If an attempt is made to calculate the fields close to the aperture, the results are inaccurate. If the distance from the point to the aperture goes to zero, the fields become infinite, whereas they can never in fact become larger (barring reflections) than the external field. It can be shown, in fact, that the actual field strength in the plane of the aperture (as distinct from the strength of the equivalent dipole) will be half the field strength that would exist if the aperture were not there.

To maintain a little more numerical accuracy near the aperture, the approximations made by Equations 74 and 75 will not be made, but Equation 73 will instead be expanded by a power series.

DIPOLE ORIENTED ALONG Y AXIS

In Figure 37:

$$r_1 = \sqrt{X^2 + Z^2 + \left(Y - \frac{\ell_2}{2}\right)^2} \tag{80}$$

$$r_2 = \sqrt{X^2 + Z^2 + \left(Y + \frac{\ell_2}{2}\right)^2} \tag{81}$$

or:

$$r_1 = (C_2 - Y\ell_2)^{1/2} \tag{82}$$

$$r_2 = (C_2 + Y\ell_2)^{1/2} \tag{83}$$

where:

$$C_2 = X^2 + Y^2 + Z^2 + \left(\frac{\ell_2}{2}\right)^2$$

Thus:

$$M = K_2 \left[(C_2 - Y\ell_2)^{-1/2} - (C_2 + Y\ell_2)^{-1/2}\right] \tag{84}$$

Expanding by the binomial theorem and combining like terms:

$$\begin{aligned} M = K_2 \Big[& b_0 C_2^{-1/2} + b_1 (Y\ell_2) C_2^{-3/2} + b_2 (Y\ell_2)^2 C_2^{-5/2} \\ & + b_3 (Y\ell_2)^3 C_2^{-7/2} + \text{--------} \\ & - b_0 C_2^{-1/2} + b_1 (Y\ell_2) C_2^{-3/2} - b_2 (Y\ell_2)^2 C_2^{-5/2} \\ & + b_3 (Y\ell_2)^3 C_2^{-7/2} + \text{--------} \Big] \end{aligned} \tag{85}$$

where:

$$b_0 = 1$$

$$b_1 = \frac{1}{2}$$

$$b_2 = \frac{1}{2}\cdot\frac{3}{2}\cdot\frac{1}{2!} = \frac{3}{8}$$

$$b_3 = \frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}\cdot\frac{1}{3!} = \frac{15}{48} = \frac{5}{16}$$

$$b_4 = \frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}\cdot\frac{7}{2}\cdot\frac{1}{4!} = \frac{105}{384}$$

$$b_5 = \frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}\cdot\frac{7}{2}\cdot\frac{9}{2}\cdot\frac{1}{5!} = \frac{840}{3840} = \frac{63}{256}$$

$$b_6 = \frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}\cdot\frac{7}{2}\cdot\frac{9}{2}\cdot\frac{11}{2}\cdot\frac{1}{6!} = \frac{10395}{46080}$$

$$b_7 = \frac{135135}{645120} = \frac{3003}{14336}$$

Thus:

$$M = 2K_2\left[\frac{b_1(Y\ell_2)}{C_2^{3/2}} + \frac{b_3(Y\ell_2)^3}{C_2^{7/2}} + \frac{b_5(Y\ell_2)^5}{C_2^{11/2}} + \frac{b_7(Y\ell_2)^7}{C_2^{15/2}} + \text{--------}\right] \tag{86}$$

The gradient in the X direction (due to the external field in the Y direction) is:

$$H_{x|y|} = -\frac{\partial M}{\partial X} \tag{87}$$

$$\frac{\partial M}{\partial X} = 2K_2\left[-\frac{b_1(Y\ell_2)C_2^{1/2}(3/2)(2X)}{C_2^{6/2}} - \frac{b_3(Y\ell_2)^3C_2^{5/2}(7/2)(2X)}{C_2^{14/2}} - \frac{b_5(Y\ell_2)^5C_2^{9/2}(11/2)(2X)}{C_2^{22/2}} - \frac{b_7(Y\ell_2)^7C_2^{13/2}(15/2)(2X)}{C_2^{30/2}} \text{--------}\right] \tag{88}$$

$$H_{x|y|} = K_2\left[\frac{6\,b_1(Y\ell_2)X}{C_2^{5/2}} + \frac{14\,b_3(Y\ell_2)^3X}{C_2^{9/2}} + \frac{22\,b_5(Y\ell_2)^5X}{C_2^{13/2}} + \frac{30\,b_7(Y\ell_2)^7X}{C_2^{17/2}} + \text{--------}\right] \tag{89}$$

The gradient in the Y direction (again due to the external field in the Y direction) is:

$$H_{y|y|} = -\frac{\partial M}{\partial Y} \tag{90}$$

$$\frac{\partial M}{\partial Y} = 2K_2 \left[\frac{C_2^{3/2}(b_1)\ell_2 - b_1(Y\ell_2)(3/2)C_2^{1/2}(2Y - \ell_2)}{C_2^{6/2}} \right.$$

$$+ \frac{C_2^{7/2}(b_3)(2Y^2\ell_2^3) - b_3(Y\ell_2)^3(7/2)C_2^{5/2}(2Y - \ell_2)}{C_2^{14/2}}$$

$$+ \frac{C_2^{11/2}(b_5)(5Y^4\ell_2^5) - b_5(Y\ell_2)^5(11/2)C_2^{9/2}(2Y - \ell_2)}{C_2^{22/2}} \tag{91}$$

$$+ \frac{C_2^{15/2}(b_7)(7Y^6\ell_2^7) - b_7(Y\ell_2)^7(15/2)C_2^{13/2}(2Y - \ell_2)}{C_2^{30/2}}$$

$$\left. + \text{--------} \right]$$

$$H_Y\Big|_y = 2K_2 \left[b_1 \frac{3(Y\ell_2)(Y - \ell_2/2) - C_2\ell_2}{C_2^{5/2}} + b_3 \frac{7(Y\ell_2)^3(Y - \ell_2/2) - 2C_2Y^2\ell_2^3}{C_2^{9/2}} \right.$$

$$+ b_5 \frac{11(Y\ell_2)^5(Y - \ell_2/2) - 5C_2Y^4\ell_2^5}{C_2^{13/2}}$$

$$+ b_7 \frac{15(Y\ell_2)^7(Y - \ell_2/2) - 7C_2Y^6\ell_2^7}{C_2^{17/2}} \tag{92}$$

$$\left. + \text{--------} \right]$$

The gradient in the Z direction is:

$$H_Z\Big|_y = -\frac{\partial M}{\partial Z} \tag{93}$$

The partial differentiation follows the same format as Equation 22; therefore:

$$H_Z\Big|_y = K_2 \left[\frac{6\,b_1(Y\ell_2)Z}{C_2^{5/2}} + \frac{14\,b_3(Y\ell_2)^3Z}{C_2^{9/2}} + \frac{22\,b_5(Y\ell_2)^5Z}{C_2^{13/2}} + \frac{30\,b_7(Y\ell_2)^7Z}{C_2^{17/2}} + \text{--------} \right] \tag{94}$$

DIPOLE ORIENTED ALONG X AXIS

Following the identical line of attack, with the dipole oriented along the X axis, yields the following relationships:

$$H_X\Big|_x = 2K_1 \left[b_1 \frac{3(X\ell_1)(X - \ell_1/2) - C_1\ell_1}{C_1^{5/2}} + b_3 \frac{7(X\ell_1)^3(X - \ell_1/2) - 2C_1X^2\ell_1^3}{C_1^{9/2}} \right.$$

$$+ b_5 \frac{11(X\ell_1)^5(X - \ell_1/2) - 5C_1X^4\ell_1^5}{C_1^{13/2}} \tag{95}$$

$$\left. + b_7 \frac{15(X\ell_1)^7(X - \ell_1/2) - 7C_1X^6\ell_1^7}{C_1^{17/2}} + \text{--------} \right]$$

$$H_Y\left(x\right) = K_1 \left[\frac{6b_1(X\ell_1)Y}{C_1^{5/2}} + \frac{14b_3(X\ell_1)^3Y}{C_1^{9/2}} + \frac{22b_5(X\ell_1)^5Y}{C_1^{13/2}} + \frac{30b_7(X\ell_1)^7Y}{C_1^{17/2}} + \text{--------}\right] \tag{96}$$

$$H_Z\left(x\right) = K_1 \left[\frac{6b_1(X\ell_1)Z}{C_1^{5/2}} + \frac{14b_3(X\ell_1)^3Z}{C_1^{9/2}} + \frac{22b_5(X\ell_1)^5Z}{C_1^{13/2}} + \frac{30b_7(X\ell_1)^7Z}{C_1^{17/2}} + \text{--------}\right] \tag{97}$$

TOTAL MAGNETIC FIELD

The total field at point P is that due to the sum of the external fields in the Y and X directions:

$$H_X = H_X\left(y\right) + H_X\left(x\right) \tag{98}$$

$$H_Y = H_Y\left(y\right) + H_Y\left(x\right) \tag{99}$$

$$H_Z = H_Z\left(y\right) + H_Z\left(x\right) \tag{100}$$

$$\begin{aligned} H_X = {} & C_1^{-5/2}K_1\ell_1\left[3X\left(X - \frac{\ell_1}{2}\right) - C_1\right] + C_2^{-5/2}K_2\ell_2[3YX] \\ & + \frac{5}{8}C_1^{-9/2}K_1\ell_1^3\left[7X^3\left(X - \frac{\ell_1}{2}\right) - 2C_1X^2\right] + \frac{5}{8}C_2^{-9/2}K_2\ell_2^3[7Y^3X] \\ & + \frac{63}{128}C_1^{-13/2}K_1\ell_1^5\left[11X^5\left(X - \frac{\ell_1}{2}\right) - 5C_1X^4\right] + \frac{63}{128}C_2^{-13/2}K_2\ell_2^5[11Y^5X] \\ & + \frac{3003}{7168}C_1^{-17/2}K_1\ell_1^7\left[15X^7\left(X - \frac{\ell_1}{2}\right) - 7C_1X^6\right] + \frac{3003}{7168}C_2^{-17/2}K_2\ell_2^7[15Y^7X] \end{aligned} \tag{101}$$

$$\begin{aligned} H_Y = {} & C_1^{-5/2}K_1\ell_1[3XY] + C_2^{-5/2}K_2\ell_2\left[3Y\left(Y - \frac{\ell_2}{2}\right) - C_2\right] \\ & + \frac{5}{8}C_1^{-9/2}K_1\ell_1^3[7X^3Y] + \frac{5}{8}C_2^{-9/2}K_2\ell_2^3\left[7Y^3\left(Y - \frac{\ell_2}{2}\right) - 2C_2Y^2\right] \\ & + \frac{63}{128}C_1^{-13/2}K_1\ell_1^5[11X^5Y] + \frac{63}{128}C_2^{-13/2}K_2\ell_2^5\left[11Y^5\left(Y - \frac{\ell_2}{2}\right) - 5C_2Y^4\right] \\ & + \frac{3003}{7168}C_1^{-17/2}K_1\ell_1^7[15X^7Y] + \frac{3003}{7168}C_2^{-17/2}K_2\ell_2^7\left[15Y^6\left(Y - \frac{\ell_2}{2}\right) - 7C_2Y^6\right] \end{aligned} \tag{102}$$

$$
\begin{aligned}
H_Z = {} & C_1^{-5/2} K_1 \ell_1 \, [3XZ] + C_2^{-5/2} K_2 \ell_2 \, [3YZ] \\
& + \frac{5}{8} C_1^{-9/2} K_1 \ell_1^3 \, [7X^3Z] + \frac{5}{8} C_2^{-9/2} K_2 \ell_2^3 \, [7Y^3Z] \\
& + \frac{63}{128} C_1^{-13/2} K_1 \ell_1^5 \, [11X^5Z] + \frac{63}{128} C_2^{-13/2} K_2 \ell_2^5 \, [11Y^5Z] \\
& + \frac{3003}{7168} C_1^{-17/2} K_1 \ell_1^7 \, [15X^7Z] + \frac{3003}{7168} C_2^{-17/2} K_2 \ell_2^7 \, [15Y^7Z]
\end{aligned}
\tag{103}
$$

Equations 101 through 103 may be placed in a format more suitable for machine calculation, as follows:

$$H_X = G_1 \times F_1 + G_2 \times F_2 - G_3 \times F_3 \tag{104}$$

$$H_Y = G_4 \times F_1 + G_5 \times F_2 - G_6 \times F_4 \tag{105}$$

$$H_Z = G_7 \times F_1 + G_8 \times F_2 \tag{106}$$

where:

$$G_1 = \frac{3K_1 (X - \ell/2)}{C_1^{1.5}}$$

$$G_2 = \frac{3K_2 X}{C_2^{1.5}}$$

$$G_3 = -\frac{K_1 \ell_1}{C_1^{1.5}}$$

$$G_4 = \frac{3K_1 Y}{C_1^{1.5}}$$

$$G_5 = \frac{3K_2 (Y - \ell_2/2)}{C_2^{1.5}}$$

$$G_6 = -\frac{K_2 \ell_2}{C_2^{1.5}}$$

$$G_7 = \frac{3K_1 Z}{C_1^{1.5}}$$

$$G_8 = \frac{3K_2 Z}{C_2^{1.5}}$$

$$F_1 = \left(\frac{X\ell_1}{C_1}\right) + 1.45833 \left(\frac{X\ell_1}{C_1}\right)^3 + 1.804688 \left(\frac{X\ell_1}{C_1}\right)^5 + 2.094727 \left(\frac{X\ell_1}{C_1}\right)^7 \cdots$$

$$F_2 = \left(\frac{Y\ell_2}{C_2}\right) + ..45833 \left(\frac{Y\ell_2}{C_2}\right)^3 + 1.804688 \left(\frac{Y\ell_2}{C_2}\right)^5 + 2.094727 \left(\frac{Y\ell_2}{C_2}\right)^7 \cdots$$

$$F_3 = 1 + 2(X\ell_1)^2 + 5(X\ell_1)^4 + 7(X\ell_1)^6 \ldots$$

$$F_4 = 1 + 2(Y\ell_2)^2 + 5(Y\ell_2)^4 + 7(Y\ell_2)^6 \ldots$$

In Figure 37, product $H\ell$ is the strength of the equivalent dipole produced in the aperture by the external magnetic field. For the portion of the field caused by the component of the field lying along the Y axis, the strength of the dipole is:

$$(H\ell)_Y = \alpha_{22} H_Y(\text{ext}) \tag{107}$$

and for the dipole lying along the X axis, the strength is:

$$(H\ell)_X = \alpha_{11} H_X(\text{ext}) \tag{108}$$

The dipole moment is appropriate for use in the classical dipole formulations based on Equation 86 but is not appropriate for the power series formulation used in the main text of this report. If the ℓ factor is taken in the derivations as the actual physical dimension of the aperture, then for K_1 and K_2:

$$K_1 = \frac{H_X}{4\pi} = \frac{\alpha_{11} H_X(\text{ext})}{4\pi\ell_1} \tag{109}$$

and:

$$K_2 = \frac{H_Y}{4\pi} = \frac{\alpha_{22} H_Y(\text{ext})}{4\pi\ell_2} \tag{110}$$

where ℓ_1 and ℓ_2 are the major and minor dimensions, respectively, of the elliptical aperture.

REFLECTING SURFACES

Two Parallel Plates

The problem of field penetration into the region between two parallel plates is of considerable interest, because it applies to the degradation of shield integrity caused by the presence of small apertures. The preceding analysis may be extended to two parallel plates, one having an aperture and the other continuous, by using image theory.

The image of the electric dipole moment is colinear with the dipole vector; however, the image of the magnetic dipole is antiparallel with the magnetic dipole vector. Taking this into consideration, a doubly infinite array of images is formed, as shown in Figure 39. The field components at a particular point in space may be obtained by an algebraic addition of all of the contributions from the aperture dipoles and the image dipoles.

Multiple Reflecting Surfaces

In principle, a rectangular area or volume behind the aperture could be formed by two or four additional reflecting surfaces, as shown in Figure 40.

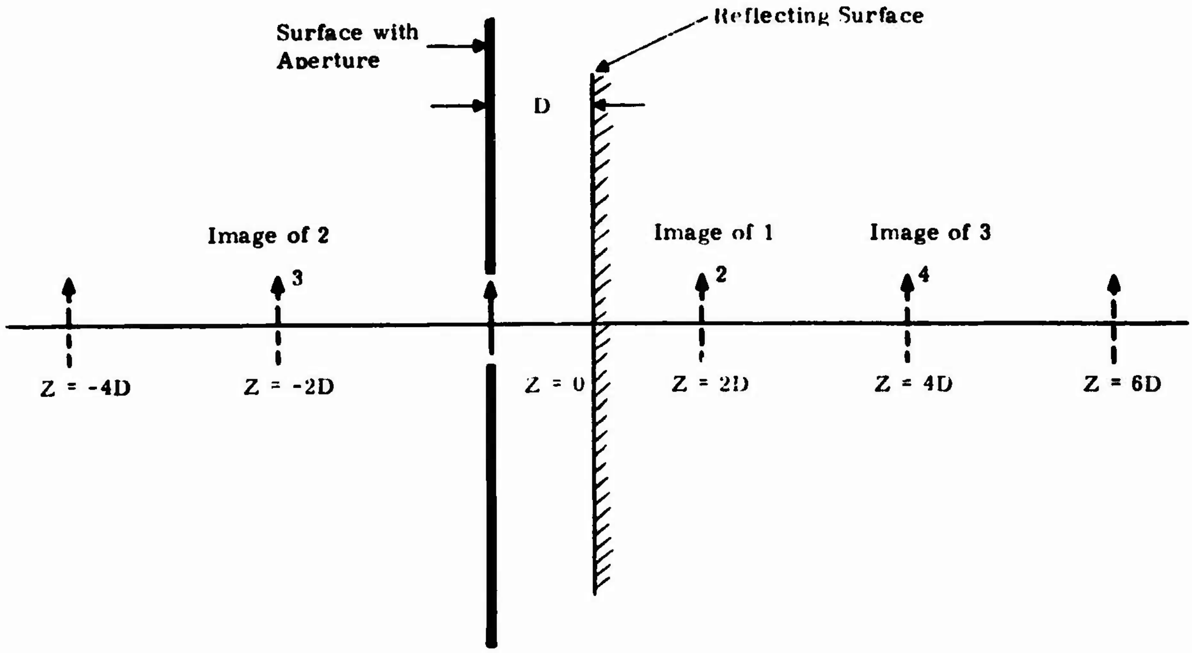


Figure 39. Reflecting Surface

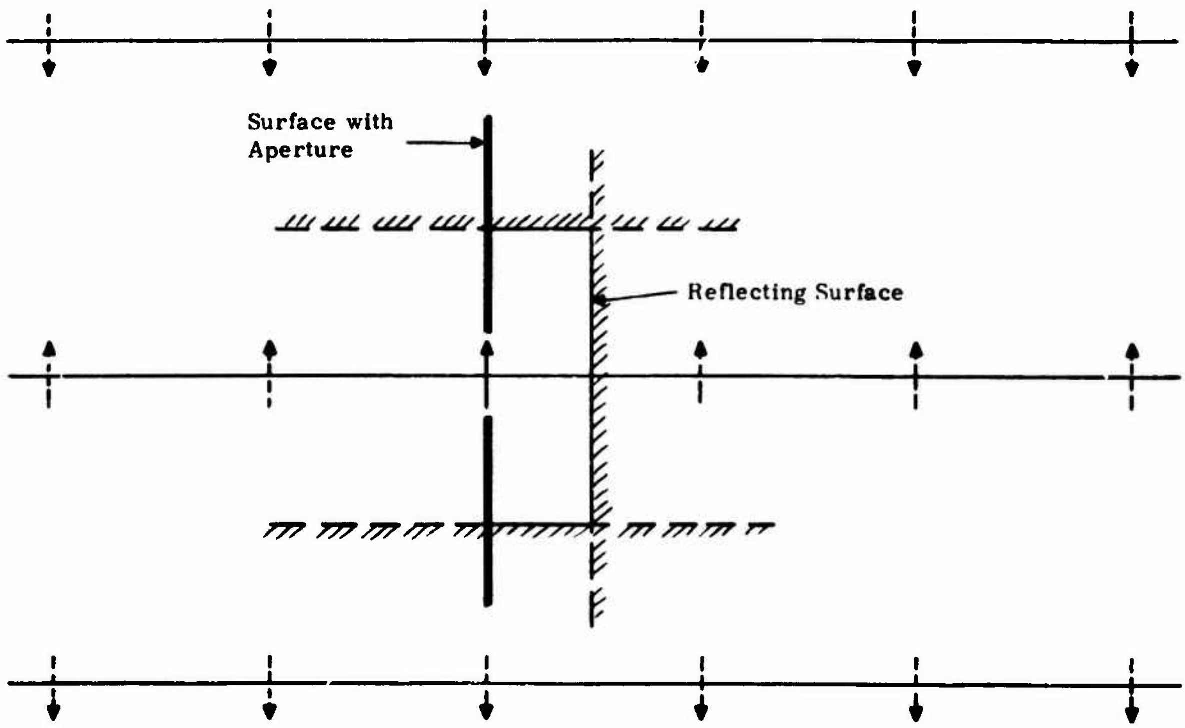


Figure 40. Multiple Reflecting Surfaces

Although the number of images increases greatly, only the images fairly near the surface generally need to be considered. Multiple reflecting surfaces have not been incorporated into this program.

FLUX LINKING A LOOP

Figure 41 shows a loop defined by four points in spaces P1 through P4, all of the points being assumed to lie in the indicated plane. At some point PL within this loop there will be a magnetic flux vector, $\overline{H}$. The XYZ components of this vector may be determined from the previous equations. The component of $\overline{H}$ that is normal to the plane is that part parallel to the normal vector, $\overline{N}$, at point PL, or:

$$H_N = \overline{H} \cos \alpha \qquad (111)$$

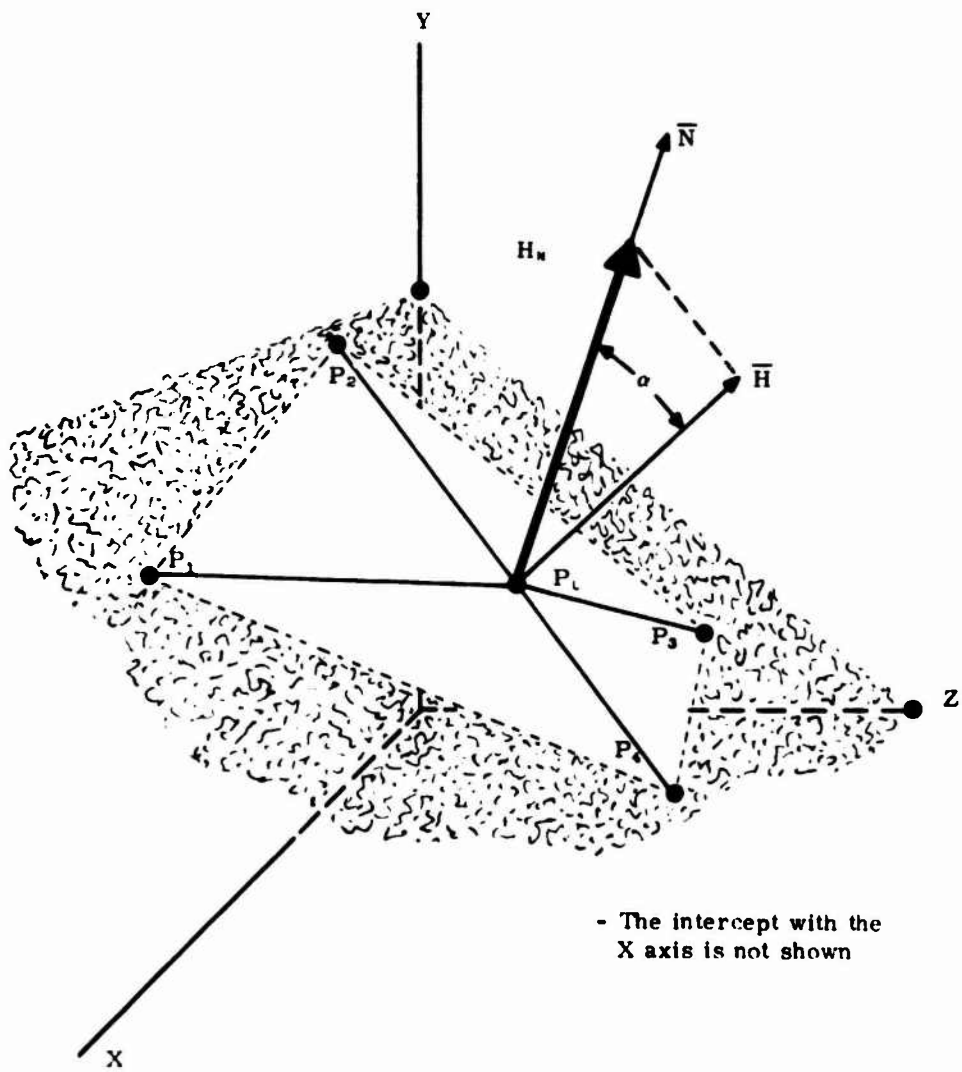


Figure 41. Flux Linking Arbitrary Four Sided Loop

which is equivalent in vector notation to the dot product:

$$H_N = \overline{H} \cdot \overline{NU} \tag{112}$$

where $\overline{NU}$ is the unit vector normal to the plane defined by points P1 through P4.

Points P1, P2, and P3 will be used to define the plane in which all of the points are assumed to lie. Two vectors that define the plane, $\overline{P1\,P2}$ and $\overline{P2\,P3}$, are then:

$$\overline{P1\,P2} = (X_{P2} - X_{P1})i + (Y_{P2} - Y_{P1})j + (Z_{P2} - Z_{P1})k \tag{113}$$

$$\overline{P2\,P3} = (X_{P3} - X_{P2})i + (Y_{P3} - Y_{P2})j + (Z_{P3} - Z_{P2})k \tag{114}$$

The normal to the plane defined by these vectors is given by the cross product:

$$\overline{N} = \overline{P1\,P2} \times \overline{P2\,P3} \tag{115}$$

which in matrix notation is:

$$\overline{N} = \begin{vmatrix} i & j & k \\ (X_{P2} - X_{P1}) & (Y_{P2} - Y_{P1}) & (Z_{P2} - Z_{P1}) \\ (X_{P3} - X_{P2}) & (Y_{P3} - Y_{P2}) & (Z_{P3} - Z_{P2}) \end{vmatrix} \tag{116}$$

or:

$$\overline{N} = \begin{vmatrix} i & j & k \\ X_{21} & Y_{21} & Z_{21} \\ X_{32} & Y_{32} & Z_{32} \end{vmatrix} \tag{117}$$

where X_{21}, Y_{21} ... Z_{32} are the corresponding quantities in Equation 116. Expanding the determinant in Equation 117 gives:

$$\begin{aligned} \overline{N} = {} & (Y_{21} Z_{32} - Y_{32} Z_{21})i \\ & - (X_{21} Z_{32} - X_{32} Z_{21})j \\ & + (X_{21} Y_{32} - X_{32} Y_{21})k \end{aligned} \tag{118}$$

The unit vector normal to the plane will be:

$$\overline{NU} = NUX\,i + NUY\,j + NUZ\,k \tag{119}$$

where:

$$\begin{aligned} NUX &= (Y_{21} Z_{32} - Y_{32} Z_{21})/NU \\ NUY &= (X_{21} Z_{32} - X_{32} Z_{21})/NU \\ NUZ &= (X_{21} Y_{32} - X_{32} Y_{21})/NU \\ NU^2 &= (Y_{21} Z_{32} - Y_{32} Z_{2})^2 + (X_{21} Z_{32} - X_{32} Z_{21})^2 + (X_{21} Y_{32} - X_{32} Y_{21})^2 \end{aligned}$$

NUMERICAL INTEGRATION OF FLUX DENSITY

The total magnetic flux normal to the loop is determined by a numerical integration process. The process is shown in Figure 42. The plane is divided vertically and horizontally into 12 equally spaced strips. For this discussion, vertical will mean the direction of point 1 to point 2 or point 4 to point 3, and horizontal will mean the direction of point P1 to point P4 or point P2 to point P3.

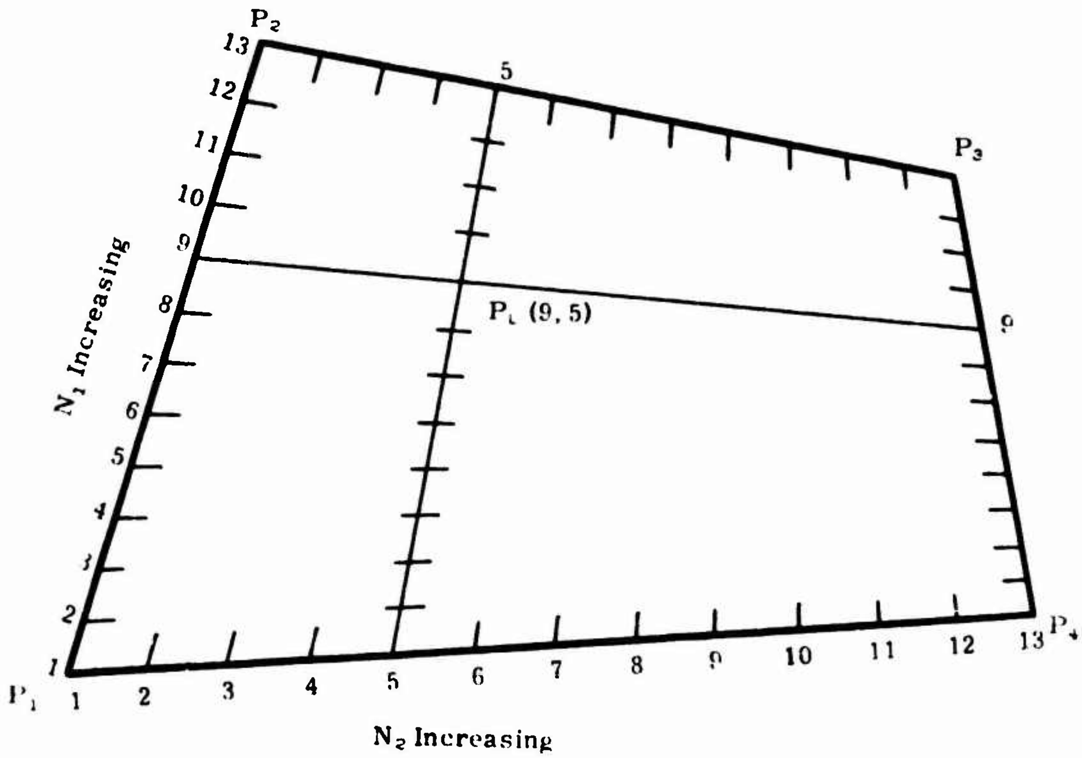


Figure 42. Integration Techniques for Flux in Plane

The 12 strips define 13 lines vertically, the intersections of which define 169 points (13 x 13), of which point PL (9, 5) is shown. The flux density at each point is evaluated and then integrated numerically along each of the vertical strips. The resultant 13 values are integrated horizontally to obtain the total magnetic flux linking the plane. The integration process used is called Weddle's rule (Ref. 11) and is based on fitting a sixth order polynomial to the array of points and then integrating the resultant polynomial. The result is:

$$\Sigma H \cdot \Delta X = \frac{3}{10} \Delta X \left(H_1 + 5H_2 + H_3 + 6H_4 + H_5 + 5H_6 + H_7 \right) \qquad (120)$$

In program APERTURE, two such polynomials are fitted to the 13 points, giving the following coefficients:

1, 5, 1, 6, 1, 5, 2, 5, 1, 6, 1, 5, 1

DEVELOPMENT OF COMPUTER PROGRAM

PROGRAM DESCRIPTION

A program listing for APERTURE is given in Figure 43. The listing shown consists of the MAIN portion of the program and two subroutines: SHAPEFAC, which is used once during the running of MAIN; and MAGFLD, which is used repetitively during the running of MAIN. Figure 44 is an elementary flow chart for MAIN.

The program first reads the input data from a file, the name of which will be requested during the program. It next determines the effective dipole moments presented by the aperture, in both the X and Y directions. X and Y are taken to be oriented along the major and minor axes respectively of the aperture.

The program next tabulates the magnetic field intensity at the desired points of the region beyond the aperture. This tabulation may or may not include the effects of a reflecting surface behind the aperture. This portion of MAIN uses the subroutine MAGFLD to calculate the field intensities at the point under consideration. Should the tabulation of field intensities not be desired, this portion of the program is bypassed.

The program then goes on toward the calculation of the flux that passes through a four-sided loop. The loop is defined by the X, Y, Z coordinates of the four points making its corners. The first three points are used to define the plane of the loop; the fourth point is assumed to lie in this same plane. After reading the coordinates of the defining points, the program calculates the field intensity at 169 points over the surface of that loop. A numerical integration of the field intensity at these 169 points is then performed to obtain the total field intensity and total flux passing through the loop. After completing the calculation of total flux through the first loop, the program reads the coordinates of additional points, and calculates the flux through such planes as may be defined. The program continues to run in this manner until no further loops are encountered. If desirable, this portion of the program also may be bypassed.

Input data -- long form -- for the APERTURE program are shown in Figure 45; Figure 46 is the short form.

MAIN Program

Before starting the detailed description of the MAIN program, the user should refer to Figure 47, which gives the terminology by which the aperture, the external magnetic field, and the point under consideration are described. The X, Y, Z coordinates of both the aperture and the point under consideration are given with respect to a reference set of axes. The plane containing the

*This listing is for a program that will be run on the General Electric Time Sharing computer. A program listing for the CDC6600 machine is included in Appendix III, "Program Listings for CDC6600 Computer."

```
1000C APERTURE-----A PROGRAM THAT CALCULATES THE MAGNETIC FIELD THAT
1010C PASSES THROUGH AN APERTURE.  FA FISHER  BLDG 9-209
1020C GENERAL ELECTRIC COMPANY 100 WOODLAWN AVE PITTSFIELD,MASS 01201
1030C PHONE (413)-494-4380
1040C DEVELOPED UNDER CONTRACT F33611-74-C-3068  USAF FLIGHT DYNAMICS LB
1050C THE PROGRAM READS DATA FROM AN EXTERNAL FILE, THE NAME OF WHICH
1060C WILL BE REQUESTED DURING EXECUTION. THE INPUT DATA FILE SHOULD
1070C BE CONSTRUCTED AS FOLLOWS:
1080C
1090C   LINE NUMBER 10 XA,YA,ZA
1100C               20 L1,L2,ANAH
1110C               30 HEXT,ANGH
1120C               40 D1,D2
1130C               50 D3
1140C               60 ZPA,ZPB,ZPC
1150C               70 YPA,YPB,YPC
1160C               80 XPA,XPB,XPC
1170C               90 D4
1180C              100 D5
1190C              110 PX1,PY1,PZ1,PX2,PY2,PZ2
1200C              120 PX3,PY3,PZ3,PX4,PY4,PZ4
1210C
1220C  (LINE NUMBERS NEED NOT BE IDENTICAL TO THOSE ABOVE)
1230C
1240C  XA,YA,ZA ARE THE COORDINATES IN METERS OF THE CENTER OF THE
1250C  APERTURE. IT IS LOCATED IN A PLANE PARALLAL TO THE XY PLANE
1260C
1270C L1 AND L2 ARE THE LENGTHS IN METERS OF THE AXES OF THE ELLIPTICAL
1280C APERTURE.  L1=MAJOR AXIS AND > L2=MINOR AXIS.
1290C  ANAH IS THE ANGLE THAT THE MAJOR AXIS OF THE APERTURE MAKES WITH
1300C  THE X AXIS.  0 DEGREES IS PARALLEL THE THE POSITIVE X AXIS.
1310C
1320C HEXT IS THE STRENGTH IN AMPERES PER METER OF THE EXTERNAL FIELD
1330C
1340C ANGH WITH RESPECT TO THE X AXIS. 0 DEGREES=PARALLEL TO X-AXIS.
1350C D1=1=YES-THERE IS A REFLECTING SURFACE PARALLEL TO THE APERTURE.
1360C D1=0=NO REFLECTING SURFACE.
1370C
1380C D2=Z COORDINATE OF THE REFLECTING SURFACE.  ENTER DUMMY VALUE IF
1390C D1=0.
1400C
1410C D3=1=YES-CALCULATE THE FIELDS OVER A PRESCRIBED VOLUME INSIDE.
1420C D3=0=NO-SKIP THIS CALCULATION.
1430C
1440C ZPA=Z COORDINATE AT WHICH CALCULATION SHOULD START
```

Figure 43. APERTURE Program Listing for the General Electric Time Sharing Computer (Sheet 1 of 9)

```
1450C ZPB=Z COORDINATE AT WHICH CALCULATION SHOULD END
1460C ZPC=Z INCREMENT SIZE
1470C YPA,YPB,YPC,XPA,XPB,XPC ARE SIMILAR FOR X AND Y COORDINATES
1480C ENTER DUMMY VALUES IF D3=0
1490C
1500C D4=1=TABULATE FIELD IN SPHERICAL COORDINATES.
1510C D4=0=TABULATE IN RECTANGULAR COORDINATES.
1520C
1530C D5=1=YES-CALCULATE THE FLUX LINKING A LOOP
1540C D5=0=NO-SKIP THIS CALCULATION.
1550C
1560C PX1,PY1,----PY4,PZ4 ARE THE COORDINATES OF FOUR POINTS THAT
1570C DEFINE THE LOOP.  THEY MUST GO AROUND THE LOOP IN CONSUCUTIVE
1580C ORDER.  ADDITIONAL LOOPS MAY BE DEFINED BY ADDITIONAL DATA IN
1590C THE SAME FORMAT.  DUMMY VALUES ARE NOT REQUIRED IF D5=0
1600C ------------------------------------
1610 FILENAME INFILE
1620 REAL L1,L2,NU1,NU2,NU3,NU,NUX,NUY,NUZ
1630 DIMENSION HN(13,13)
1640 DIMENSION TB6A(13)
1650 DIMENSION PATHA(13)
1660 10 PRINT 15
1670 15 FORMAT(" INPUT FILE NAME")
1680 20 INPUT,INFILE
1690 30 PRINT 115
1700C CARRIAGE CONTROL FORMAT STATEMENTS
1710 110 FORMAT(1H-)
1720 115 FORMAT(1H0)
1730 120 FORMAT(1H )
1740 122 FORMAT(1H&)
1750 123 FORMAT(1H+)
1760C OUTPUT DATA FORMATS
1770 130 FORMAT(6E12.3)
1780C DATA HEADING FORMATS
1790 140 FORMAT(" APERTURE COORDINATES--X=",1E12.3,"  METERS")
1800 145 FORMAT("                        Y=",1E12.3,"  METERS")
1810 150 FORMAT("                        Z=",1E12.3,"  METERS")
1820 155 FORMAT(" APERTURE DIMENSIONS--MAJOR AXIS=",1E12.3,"  METERS")
1830 160 FORMAT("                      MINOR AXIS=",1E12.3,"  METERS")
1840 165 FORMAT(" APERTURE INCLINED",1E12.3,"  DEGREES FROM X AXIS")
1850 170 FORMAT(" EXTERNAL MAGNETIC FIELD=",1E12.3,"  AMPERES PER METER"
1860 175 FORMAT(" AND INCLINED",1E12.3,"  DEGREES FROM THE X AXIS")
1870 180 FORMAT(" THERE IS NO REFLECTING SURFACE")
1880 185 FORMAT(" THERE IS A REFLECTING SURFACE LOCATED AT Z=",1E12.3,
1890& "  METERS")
1900 188 FORMAT(" LOOP NUMBER  ",I5)
1910 190 FORMAT(" LOOP AREA=",1E12.3,"  SQUARE METERS")
1920 192 FORMAT(" TOTAL FLUX=",1E12.3,"  WEBERS")
```

Figure 43. APERTURE Program Listing for the General Electric Time Sharing Computer (Sheet 2 of 9)

```
1930 195 FORMAT(" OUT OF DATA")
1940 200 FORMAT(V)
1950 210 FORMAT(" POINT      X              Y              Z")
1960 220 FORMAT(I5,3E12.3)
1970 READ(INFILE,200,END=1960)LINE,XA,YA,ZA
1980 READ (INFILE,200,END=1960)LINE,L1,L2,ANGA
1990 PRINT 140,XA
2000 PRINT 145,YA
2010 PRINT 150,ZA
2020 PRINT 155,L1
2030 PRINT 160,L2
2040 PRINT 165,ANGA
2050 READ(INFILE,200,END=1960)LINE,HEXT,ANGH
2060 PRINT 115
2070 PRINT 170,HEXT
2080 PRINT 175,ANGH
2090 PRINT 115
2100 READ(INFILE,200,END=1960)LINE,D1,D2
2110 275 IF(D1)280,280,290
2120 280 PRINT 180
2130 PRINT 115
2140 285 GOTO295
2150 290 PRINT 185,D2
2160 PRINT 115
2170 295 CONTINUE
2180 READ(INFILE,200,END=1960)LINE,D3
2190 READ(INFILE,200,END=1960)LINE,ZPA,ZPB,ZPC
2200 READ(INFILE,200,END=1960)LINE,YPA,YPB,YPC
2210 READ(INFILE,200,END=1960)LINE,XPA,XPB,XPC
2220 READ(INFILE,200,END=1960)LINE,D4
2230 PI=3.14159265
2240 CALL SHAPEFAC(L1,L2,A11,A22)
2250 IF(D3)1950,1950,2200
2260 2200 IF(D4)2201,2201,2206
2270 2201 PRINT 2202
2280 2202 FORMAT("   X              Y              Z              H-X
2290&            H-Y            H-Z")
2300 GOTO 2204
2310 2206 PRINT 2207
2320 2207 FORMAT("   X              Y              Z              HTOT
2330&            LAT            LONG")
2340 2208 PRINT 120
2350 GOTO 2450
2360 2204 PRINT 120
2370 2450 CONTINUE
2380 J1=IFIX((ZPB-ZPA)/ZPC)+1
2390 J2=IFIX((YPB-YPA)/YPC)+1
2400 J3=IFIX((XPB-XPA)/XPC)+1
```

Figure 43. APERTURE Program Listing for the General Electric Time Sharing Computer (Sheet 3 of 9)

```
2410 DO1950 I3=1,J3,1
2420 DO1950 I2=1,J2,1
2430 DO1950 I1=1,J1,1
2440 XP1=XPA+(I3-1)*XPC
2450 YP1=YPA+(I2-1)*YPC
2460 ZP1=ZPA+(I1-1)*ZPC
2470 1002 CONTINUE
2480 1199 CONTINUE
2490 1750 CALL MAGFLD(ANGA,ANGH,XP1,YP1,ZP1,XA,YA,ZA,HEXT,A11,A22,
2500& HPX1,HPY1,HPZ1,L1,L2,D1,D2)
2510 3370 IF(D4)1210,1210,4000
2520 1210 PRINT 1220,XP1,YP1,ZP1,HPX1,HPY1,HPZ1
2530 GOTO 1950
2540 1220 FORMAT(6E12.3)
2550 4000 D=SQRT(HPX1*HPX1+HPZ1*HPZ1)
2560 4002 IF(ABS(HPY1)-ABS(D)) 4004,4004,4010
2570 4004 ANG1=90-57.2957795*(ATAN(HPY1/D))
2580 4006 GOTO 4012
2590 4010 ANG1=57.2957795*(ATAN(D/HPY1))
2600 4012 IF(ABS(HPX1)-ABS(HPZ1)) 4014,4014,4020
2610 4014 ANG2=90-57.2957795*(ATAN(HPX1/HPZ1))
2620 4016 GOTO 4030
2630 4020 ANG2=57.2957795*(ATAN(HPZ1/HPX1))
2640 4030 IF(HPY1) 4050,4050,4040
2650 4040 GOTO 4110
2660 4050 ANG1=180-ANG1
2670 4110 CONTINUE
2680 4120 IF(HPX1)4180,4130,4130
2690 4130 IF(HPZ1)4160,4140,4140
2700 4140 ANG2=ANG2
2710 4150 GOTO 4215
2720 4160 ANG2=-ANG2
2730 4170 GOTO 4215
2740 4180 IF(HPZ1)4210,4190,4190
2750 4190 ANG2=180-ANG2
2760 4200 GOTO 4215
2770 4210 ANG2=-(180-ANG2)
2780 4215 CONTINUE
2790 HPT=SQRT(HPX1*HPX1+HPY1*HPY1+HPZ1*HPZ1)
2800 4230 PRINT 1220,XP1,YP1,ZP1,HPT,ANG1,ANG2
2810 1950 CONTINUE
2820 PRINT 110
2830 READ(INFILE,200,END=1960)LINE,D5
2840 2100 IF(D5)1400,1400,1955
2850 1955 CONTINUE
2860 JX=0
2870 1957 CONTINUE
2880 READ(INFILE,200,END=1960)LINE,PX1,PY1,PZ1,PX2,PY2,PZ2
```

Figure 43. APERTURE Program Listing for the General Electric Time Sharing Computer (Sheet 4 of 9)

```
2890 READ(INFILE,200,END=1960)LINE,PX3,PY3,PZ3,PX4,PY4,PZ4
2900C THESE ARE THE SIDES OF THE QUADRILATERAL
2910 2110 CONTINUE
2920 X21=PX2-PX1
2930 X32=PX3-PX2
2940 X43=PX4-PX3
2950 X14=PX1-PX4
2960 Y21=PY2-PY1
2970 Y32=PY3-PY2
2980 Y43=PY4-PY3
2990 Y14=PY1-PY4
3000 Z21=PZ2-PZ1
3010 Z32=PZ3-PZ2
3020 Z43=PZ4-PZ3
3030 Z14=PZ1-PZ4
3040C THIS IS A DIAGONAL OF THE QUADRILATERAL
3050 X31=PX3-PX1
3060 Y31=PY3-PY1
3070 Z31=PZ3-PZ1
3080 T21=SQRT(X21*X21+Y21*Y21+Z21*Z21)
3090 T32=SQRT(X32*X32+Y32*Y32+Z32*Z32)
3100 T43=SQRT(X43*X43+Y43*Y43+Z43*Z43)
3110 T14=SQRT(X14*X14+Y14*Y14+Z14*Z14)
3120 T31=SQRT(X31*X31+Y31*Y31+Z31*Z31)
3130 S1=(T21+T32+T31)/2
3140 A1=SQRT(S1*(S1-T21)*(S1-T32)*(S1-T31))
3150 S2=(T43+T14+T31)/2
3160 A2=SQRT(S2*(S2-T43)*(S2-T14)*(S2-T31))
3170 AREA=A1+A2
3180C THESE ARE THE MIDPOINTS OF THE ENDS OF THE QUADRILATERAL
3190 XPM1=PX1+X21/2
3200 YPM1=PY1+Y21/2
3210 ZPM1=PZ1+Z21/2
3220 XPM2=PX4-X43/2
3230 YPM2=PY4-Y43/2
3240 ZPM2=PZ4-Z43/2
3250 XPM21=XPM2-XPM1
3260 YPM21=YPM2-YPM1
3270 ZPM21=ZPM2-ZPM1
3280 TPM=SQRT(XPM21*XPM21+YPM21*YPM21+ZPM21*ZPM21)
3290C THESE ARE THE COMPONENTS OF THE NORMAL VECTOR
3300 NU1=Y21*Z32-Y32*Z21
3310 NU2=-X21*Z32+X32*Z21
3320 NU3=X21*Y32-X32*Y21
3330 NU=SQRT(NU1*NU1+NU2*NU2+NU3*NU3)
3340C THESE ARE THE COMPONONENTS OF THE UNIT NORMAL VECTOR
3350 NUX=NU1/NU
3360 NUY=NU2/NU
```

Figure 43. APERTURE Program Listing for the General Electric Time Sharing Computer (Sheet 5 of 9)

```
3370 NUZ=NU3/NU
3380 3550 CØNTINUE
3390 3560 DØ 3880 N2=1,13,1
3400 3570 DØ3880 N1=1,13,1
3410 XP5=PX1+X21*(N1-1)/12
3420 YP5=PY1+Y21*(N1-1)/12
3430 ZP5=PZ1+Z21*(N1-1)/12
3440 XP6=PX2+X32*(N2-1)/12
3450 YP6=PY2+Y32*(N2-1)/12
3460 ZP6=PZ2+Z32*(N2-1)/12
3470 XP7=PX4-X43*(N1-1)/12
3480 YP7=PY4-Y43*(N1-1)/12
3490 ZP7=PZ4-Z43*(N1-1)/12
3500 XP8=PX1-X14*(N2-1)/12
3510 YP8=PY1-Y14*(N2-1)/12
3520 ZP8=PZ1-Z14*(N2-1)/12
3530 X75=XP7-XP5
3540 Y75=YP7-YP5
3550 Z75=ZP7-ZP5
3560 X86=XP8-XP6
3570 Y86=YP8-YP6
3580 Z86=ZP8-ZP6
3590 T86=SQRT(X86*X86+Y86*Y86+Z86*Z86)
3600 XPL=XP8-X86*(N1-1)/12
3610 YPL=YP8-Y86*(N1-1)/12
3620 ZPL=ZP8-Z86*(N1-1)/12
3630 CALL MAGFLD(ANGA,ANGH,XPL,YPL,ZPL,XA,YA,ZA,HEXT,A11,A22,HPX1
3640& ,HPY1,HPZ1,L1,L2,D1,D2)
3650 HNP=HPX1*NUX+HPY1*NUY+HPZ1*NUZ
3660C THESE ARE THE HN'S AT THE VARIØUS PØINTS ØF THE QUADRILATERAL
3670 HN(N1,N2)=HNP
3680C THESE ARE THE DISTANCES TØP TØ BØTTØM ALØNG THE QUADRILATERAL
3690 T86A(N2)=T86
3700 3880 CØNTINUE
3710 3890 DØ3950 N2=1,13,1
3720 DELTA1=T86A(N2)/12
3730C THIS EVALUATES FLUX ALØNG THE LINES IN THE DIRECTIØN P1-->P2
3740C AND P4-->P3
3750 PATH=HN(1,N2)+HN(2,N2)*5+HN(3,N2)+HN(4,N2)*6+HN(5,N2)+HN(6,N2)*5+
3760& HN(7,N2)*2+HN(8,N2)*5+HN(9,N2)+HN(10,N2)*6+
3770& HN(11,N2)+HN(12,N2)*5+HN(13,N2)
3780 PATHA(N2)=0.3*DELTA1*PATH
3790 3950 CØNTINUE
3800 DELTA2=TPM/12
3810C THIS EVALUATES THE RESULTANT FLUX IN THE DIRECTIØN P1-->P4
3820C AND P2-->P3
3830 HTØT=PATHA(1)+PATHA(2)*5+PATHA(3)+PATHA(4)*6+PATHA(5)+
3840& PATHA(6)*5+PATHA(7)*2+PATHA(8)*5+PATHA(9)+PATHA(10)*6+
```

Figure 43. APERTURE Program Listing for the General Electric Time Sharing Computer (Sheet 6 of 9)

```
3850& PATHA(11)+PATHA(12)*5+PATHA(13)
3860 HTOT=0.3*DELTA2*HTOT
3870 BTOT=HTOT*4*PI*1E-7
3880 JX=JX+1
3890 4315 PRINT 188,JX
3900 PRINT 120
3904 PRINT 210
3906 PRINT 120
3910 J=1
3920 PRINT 220,J,PX1,PY1,PZ1
3930 J=2
3940 PRINT 220,J,PX2,PY2,PZ2
3950 J=3
3960 PRINT 220,J,PX3,PY3,PZ3
3970 J=4
3980 PRINT 220,J,PX4,PY4,PZ4
3990 PRINT 120
4000 4320 PRINT 190,AREA
4010 4330 PRINT 192,BTOT
4020 4340 PRINT 110
4030 GOTO 1957
4040 4360 PRINT 110
4050 1960 PRINT 195
4060 1400 STOP
4070 END
4080 SUBROUTINE SHAPEFAC(L1,L2,A11,A22)
4090 REAL L1,L2
4100 PI=3.14159265
4110 E1=1-(L2/L1)**2
4120 E2=SQRT(E1)
4130 IF(L2/L1<0)GOTO 3130
4140 IF(L2/L1>1) GOTO 3160
4150C ##################################
4160C CELI(1,E2) AND CELI(2,E2) ARE MATH LIBRARY ROUTINES THAT
4170C EVALUATE THE ELLIPTIC INTEGRALS OF THE FIRST AND SECOND KINDS.
4180 Y1=CELI(1,E2)
4190 Y2=CELI(2,E2)
4200C ##################################
4210 CON1=2*PI*(L1/2)**3/3
4220 A11=CON1*E1/(Y1-Y2)
4230 A22=CON1*E1*(1-E1)/(Y2-(1-E1)*Y1)
4240 A33=CON1*(1-E1)/Y2
4250 RETURN
4260 3130 PRINT 3140
4270 3140 FORMAT(" L2/L1 IS NEGATIVE. THIS IS AN ERROR")
4280 RETURN
4290 3160 PRINT 3170
4300 3170 FORMAT(" L2 IS LARGER THAN L1.  THIS IS AN ERROR")
```

Figure 43. APERTURE Program Listing for the General Electric Time Sharing Computer (Sheet 7 of 9)

```
4310 STOP;END
4320 SUBROUTINE MAGFLD(ANGA,ANGH,XP1,YP1,ZP1,XA,YA,ZA,HEXT,A11,
4330& A22,HPX1,HPY1,HPZ1,L1,L2,D1,D2)
4340 REAL K1,K2,K5,K6,L1,L2
4350 N=0
4360 RAD=57.2957795
4370 PI=3.14159265
4380 K1=COS(ANGA/RAD)
4390 K2=SIN(ANGA/RAD)
4400C CALCULATION OF SHIFTED COORDINATES OF POINT UNDER INVESTIGATION
4410 XP2=XP1*K1+YP1*K2
4420 YP2=-XP1*K2+YP1*K1
4430 ZP2=ZP1
4440 HPX2=0
4450 HPY2=0
4460 HPZ2=0
4470 ZAA=ZA
4480C CALCULATION OF DISTANCES FROM APERTURE TO POINT UNDER STUDY
4490 9130 XC=XP2-XA
4500 9140 YC=YP2-YA
4510 9150 ZC=ZP2-ZAA
4520 9160 C1=XC*XC+YC*YC+ZC*ZC+L1*L1/4
4530 C2=XC*XC+YC*YC+ZC*ZC+L2*L2/4
4540C CALCULATION OF H FIELD PARALLEL TO AXES OF APERTURE
4550 CON1=1.43833
4560 CON2=1.804688
4570 CON3=2.094727
4580C CALCULATION OF FIELD PARALLEL TO AXES OF APERTURE
4590 ANAH=(ANGH-ANGA)/RAD
4600 HMAJ=HEXT*COS(ANAH)
4610 HMIN=HEXT*SIN(ANAH)
4620C CALCULATION OF ROTATED COMPONENTS OF MAGNETIC FIELD
4630 K5=-A11*HMAJ/(4*PI*L1)
4640 K6=-A22*HMIN/(4*PI*L2)
4650 C3=XC*L1
4660 9165 IF(ABS(C3)-1E-5) 9180,9170,9170
4670 9170 F3=C3*C3
4680 F7=1+2*F3+5*F3*F3+7*F3*F3*F3
4690 GOTO 9190
4700 9180 F7=1
4710 9190 C5=C3/C1
4720 9195 IF(ABS(C5)-1E-5) 9210,9200,9200
4730 9200 C7=C5*C5
4740 F1=C5*(1+CON1*C7+CON2*C7*C7+CON3*C7*C7*C7)
4750 GOTO 9220
4760 9210 F1=C5
4770 9220 C4=YC*L2
4780 9230 IF(ABS(C4)-1E-5) 9250,9240,9240
```

Figure 43. APERTURE Program Listing for the General Electric Time Sharing Computer (Sheet 8 of 9)

```
4790 9240 F5=C4*C4
4800 F8=1+2*F5+5*F5*F5+7*F5*F5*F5
4810 GOTO 9260
4820 9250 F8=1
4830 9260 C6=C4/C2
4840 9270 IF(ABS(C6)-1E-5) 9300,9280,9280
4850 9280 C8=C6*C6
4860 F2=C6*(1+CON1*C8+CON2*C8*C8+CON3*C8*C8*C8)
4870 GOTO 9320
4880 9300 F2=C6
4900 9320 CONTINUE
4910 G1=3*K5*XC/C1**1.5
4920 G2=3*K6*XC/C2**1.5
4930 G3=K5*L1/C1**1.5
4940 G4=3*K5*YC/C1**1.5
4950 G5=3*K6*YC/C2**1.5
4960 G6=K6*L2/C2**1.5
4970 G7=3*K5*ZC/C1**1.5
4980 G8=3*K6*ZC/C2**1.5
4990 HPX=G1*F1+G2*F2-G3*F7
5000 HPY=G4*F1+G5*F2-G6*F8
5010 HPZ=G7*F1+G8*F2
5020 9680 IF(D1) 9760,9760,9682
5030 9682 HPX2=HPX2+HPX
5040 HPY2=HPY2+HPY
5050 HPZ2=HPZ2+HPZ
5060 9690 IF(ABS(HPX)-.005*ABS(HPX2))9700,9700,9720
5070 9700 IF(ABS(HPY)-.005*ABS(HPY2))9710,9710,9720
5080 9710 IF(ABS(HPZ)-.005*ABS(HPZ2))9770,9770,9720
5090 9720 IF(N-10)9730,9730,9770
5100 9730 N=N+1
5110 9740 ZAA=ZAA-((-1)**N)*2*N*D2
5120 GOTO9130
5130 9760 HPX2=HPX2+HPX
5140 HPY2=HPY2+HPY
5150 HPZ2=HPZ2+HPZ
5155 9770 CONTINUE
5160C CALCULATION OF COMPONENTS OF MAGNETIC FIELD ROTATED BACK
5170C TO THE REFERENCE AXES
5180 HPX1=HPX2*K1-HPY2*K2
5190 HPY1=HPX2*K2+HPY2*K1
5200 HPZ1=HPZ2
5210 RETURN
5220 5230 STOP;END
```

Figure 43. APERTURE Program Listing for the General Electric Time Sharing Computer (Sheet 9 of 9)

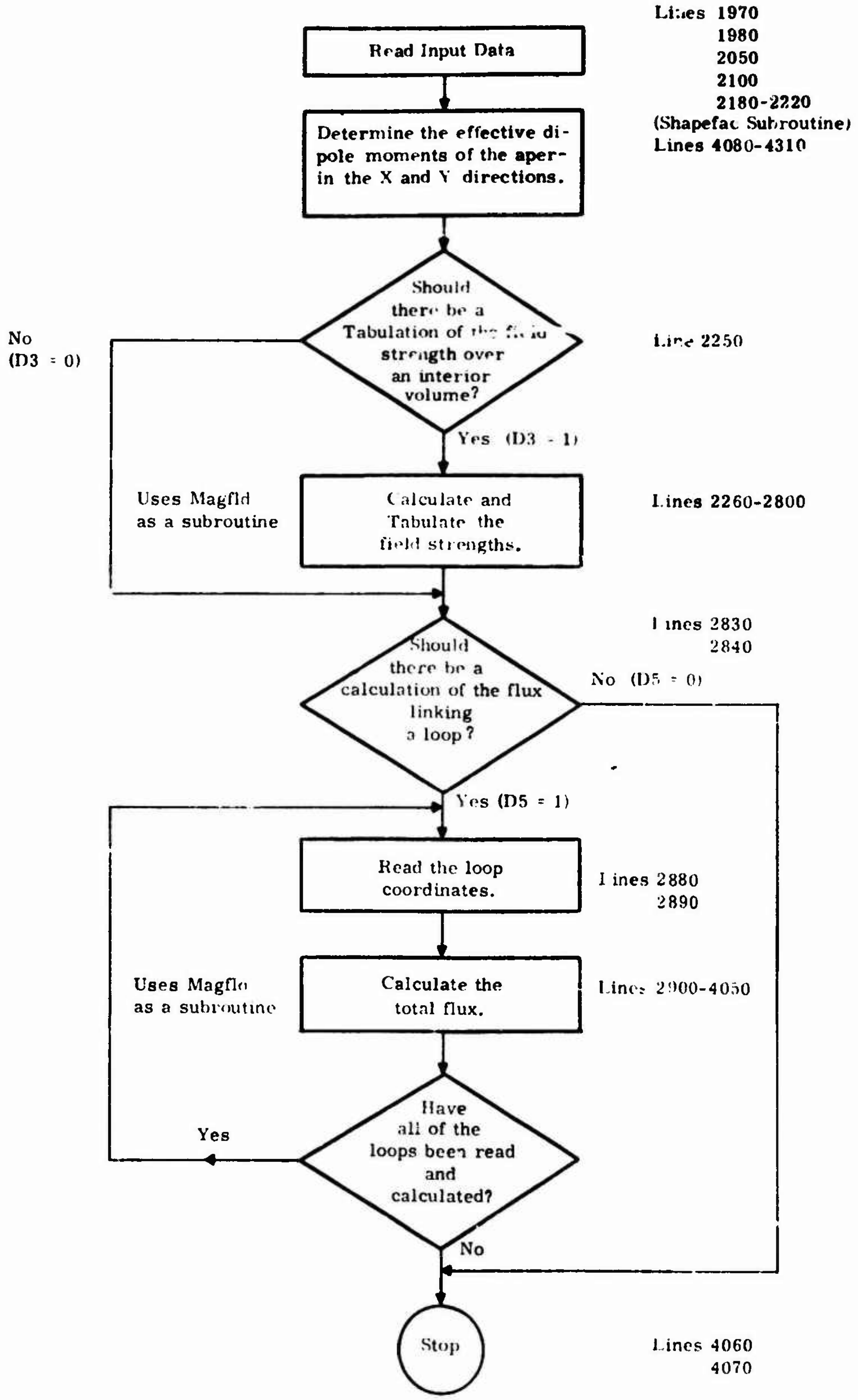


Figure 44. Main Program -- Elementary Flowchart

Coordinates of the Aperture

10b   X Coordinate, (XA)   Y Coordinate (YA)   Z Coordinate (ZA)

Dimensions and Orientation of the Aperture

20b   Major Axis Length ______, (L1)   Minor Axis Length ______, (L2)   Orientation of Major Axis with Respect to the X-Axis (ANAH) ______ (ANAH)

Magnitude and Direction of the External Magnetic Field

30b   Field Strength, (HEXT)   Orientation with Respect to X-Axis (ANGH)

Reflecting Surface

40b   Is there a reflecting surface? For Yes enter 1 (one) and for No enter 0 (zero) ______ (D1) ______,   How far from the aperture is the reflecting surface? If there is no reflecting surface enter the dummy number 1000 ______ (D2)

Tabulation of Field Strength

50b   Do you want a table of the field strengths over the interior region? For Yes enter 1 (one) and for No enter 0 (zero) ______ (D3)

Volume Over Which Fields are to be Calculated

Z Dimension

60b   a) Start at, (ZPA)   b) End at, (ZPB)   c) And increment in Steps of (ZPC) (ZPC)

Y Dimension

70b   a) Start at, (YPA)   b) End at, (YPB)   c) And Increment in Steps of, (YPC)

X Dimension

80b   a) Start at, (XPA)   b) End at, (XPB)   c) And Increment in Steps of, (XPC)

[On lines 60, 70, and 80 enter the dummy numbers 0 (zero) in each of the locations if you don't want the fields tabulated]

Output Format

90b   Do you want the tabulation in rectangular or spherical coordinates? Enter 1 (one) for spherical or 0 (zero) for rectangular. Enter a dummy value if no tabulation is desired. ______ (D4)

Total Flux In a Loop

100b   Do you want to determine the total flux linking a loop? Enter 1 (one) for Yes and 0 (zero) for No. ______ (D5)

Enter the X, Y, Z coordinates of the four points defining the loop. They should go in sequence around the loop. All four points must lie in the same plane. If a loop calculation is not desired the following lines may be left blank, in which case the program makes an automatic stop.

Point 1   Point 2

110b   X (PX1), Y (PY1), Z (PZ1),   X (PX2), Y (PY2), Z (PZ2),

Point 3   Point 4

120b   (PX3), (PY3), (PZ3),   (PX4), (PY4), (PZ4),

Successive lines following the format of 110 and 120 may be used to enter the defining points for other loops. The program will automatically stop when it runs out of data.

Figure 45. Input Data for Program APERATURE -- Long Form

10b	________, (XA)	________, (YA)	________ (ZA)			
20b	________, (L1)	________, (L2)	________ (ANAH)			
30b	________, (HEXT)	________, (ANGH)				
40b	________, (D1)	________, (D2)				
50b	________, (D3)					
60b	________, (ZPA)	________, (ZPB)	________ (ZPC)			
70b	________, (YPA)	________, (YPB)	________ (YPC)			
80b	________, (XPA)	________, (XPB)	________ (XPC)			
90b	________, (D4)					
100b	________, (D5)					
110b	________, (PX1)	________, (PY1)	________, (PZ1)	________, (PX2)	________, (PY2)	________ (PZ2)
120b	________, (PX3)	________, (PY3)	________, (PZ3)	________, (PX4)	________, (PY4)	________ (PZ4)

Figure 46. Input Data for Program APERATURE -- Short Form

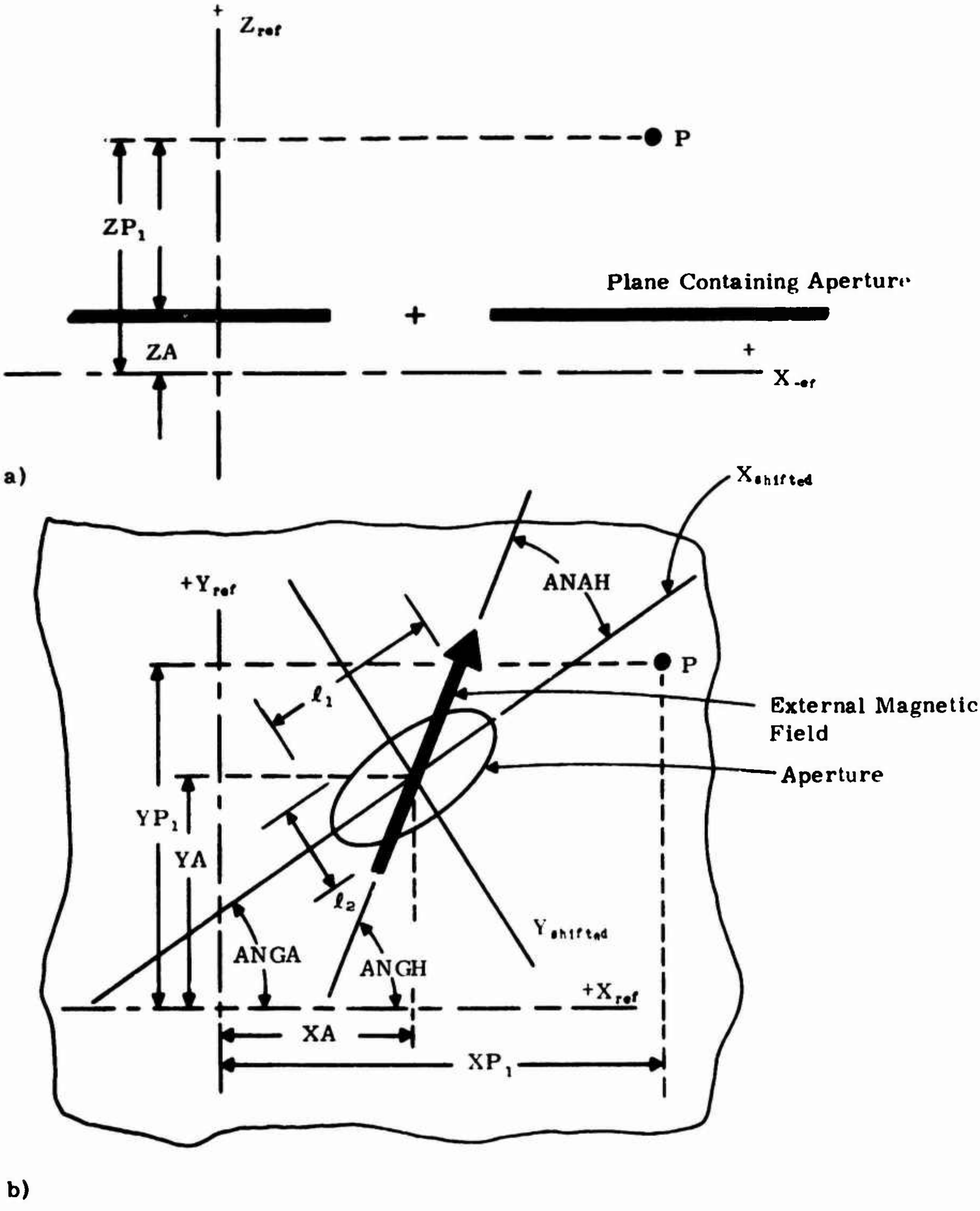


Figure 47. Descriptions of Aperture, Magnetic Field and Point Under Considerations: a) Looking Down on XZ Plane b) Looking From Outside Onto XY Plane

aperture is assumed to lie parallel to the plane defined by the reference X and Y axes. Frequently the XYZ zero point of the reference axis will be taken to coincide with the center of the aperture. While this is convenient, it is not necessary. The aperture is assumed to be an ellipse with a major axis, L1, and a minor axis, L2. The angle which the major axis makes with respect to the reference axis is called ANGA.

The coordinates of the center of the aperture -- XA, YA, and ZA -- are input quantities. Likewise, the length of the major axis, L1, and the length of the minor axis, L2, and ANGA are also input quantities.

The magnetic field which illuminates the aperture is assumed to lie in the same XY plane as that containing the aperture. The field vector is oriented at an angle ANGH to the reference X axis. The magnitude of the external magnetic field, HEXT, and the angle it makes with respect to the reference X axis, ANGH, are also input quantities.

A defined quantity used during the running of the program is the angle between the major axis of the aperture and the magnetic field vector, ANAH.

The point under consideration is defined in terms of the reference X, Y, and Z axes by the parameters XP1, YP1, and ZP1. These are not input quantities. During the running of the program, the coordinates defining the point are translated to a new set of axes, defined by the major and minor axes of the elliptical aperture. These latter are not shown on Figure 47, but go by the designations XP2, YP2, and ZP2.

A detailed flow chart of the MAIN program is given on Figure 48. The program starts with a series of comments on the program and a set of abbreviated operating instructions. These are given in lines 100-1600. They, of course, do not affect the running of the program. Definitions and dimensions of file names, variables, and arrays, are given in lines 1610-1650.

The name of the file holding the input data is given in lines 1660 and 1680. Since the program is at present configured for the General Electric time sharing system, this name is an input quantity entered on the teletypewriter. For batch processing a change will have to be made at this point.

Lines 1700-1960 are devoted to housekeeping and the setting up of formats of headings.

The coordinates defining the center of the aperture are read at line 1970. The date on the size and orientation of the aperture is read at line 1980, and data on the magnitude and orientation of the external magnetic field is read at line 2050.

The next quantity read is D1, a flag used to say whether or not there is a reflecting surface to be considered. D2 defines the location of this reflec-

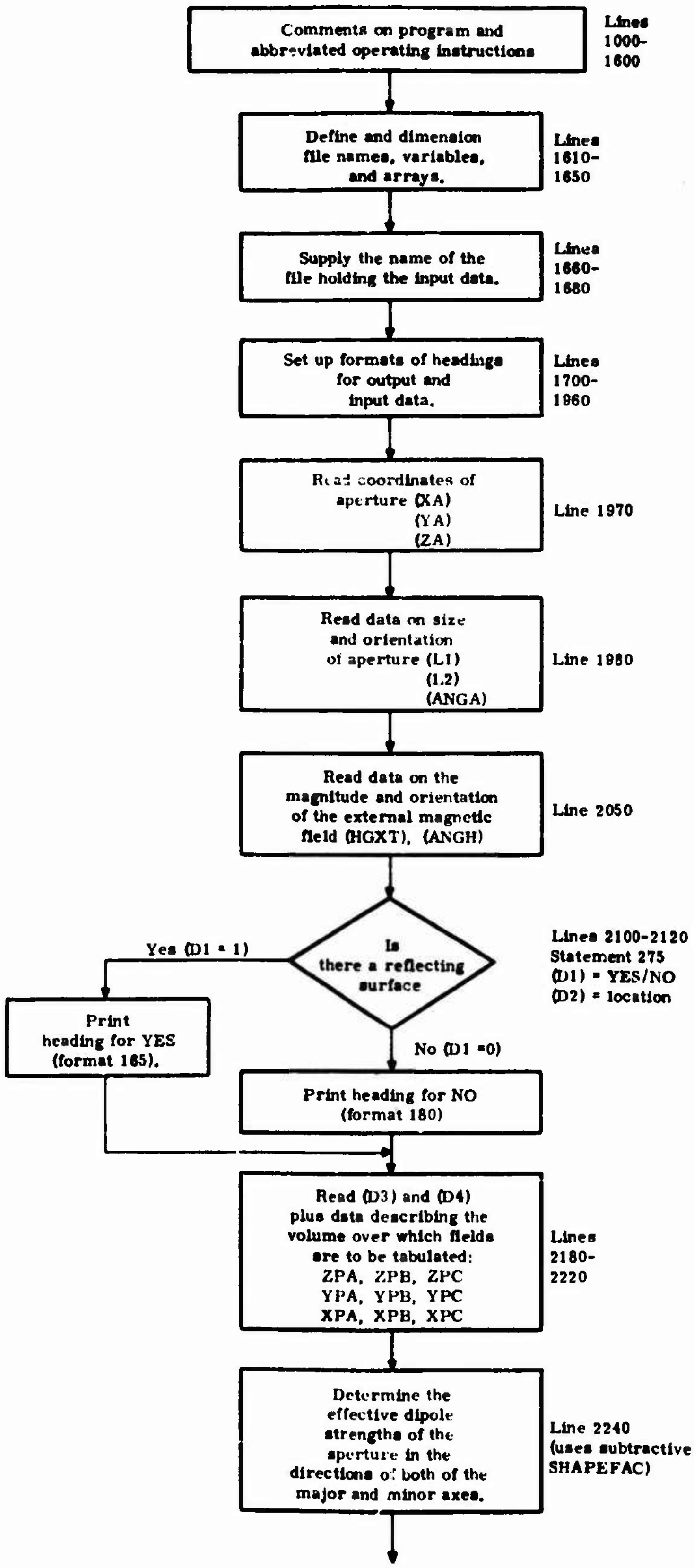


Figure 48. MAIN Program -- Detailed Flowchart (Sheet 1 of 5)

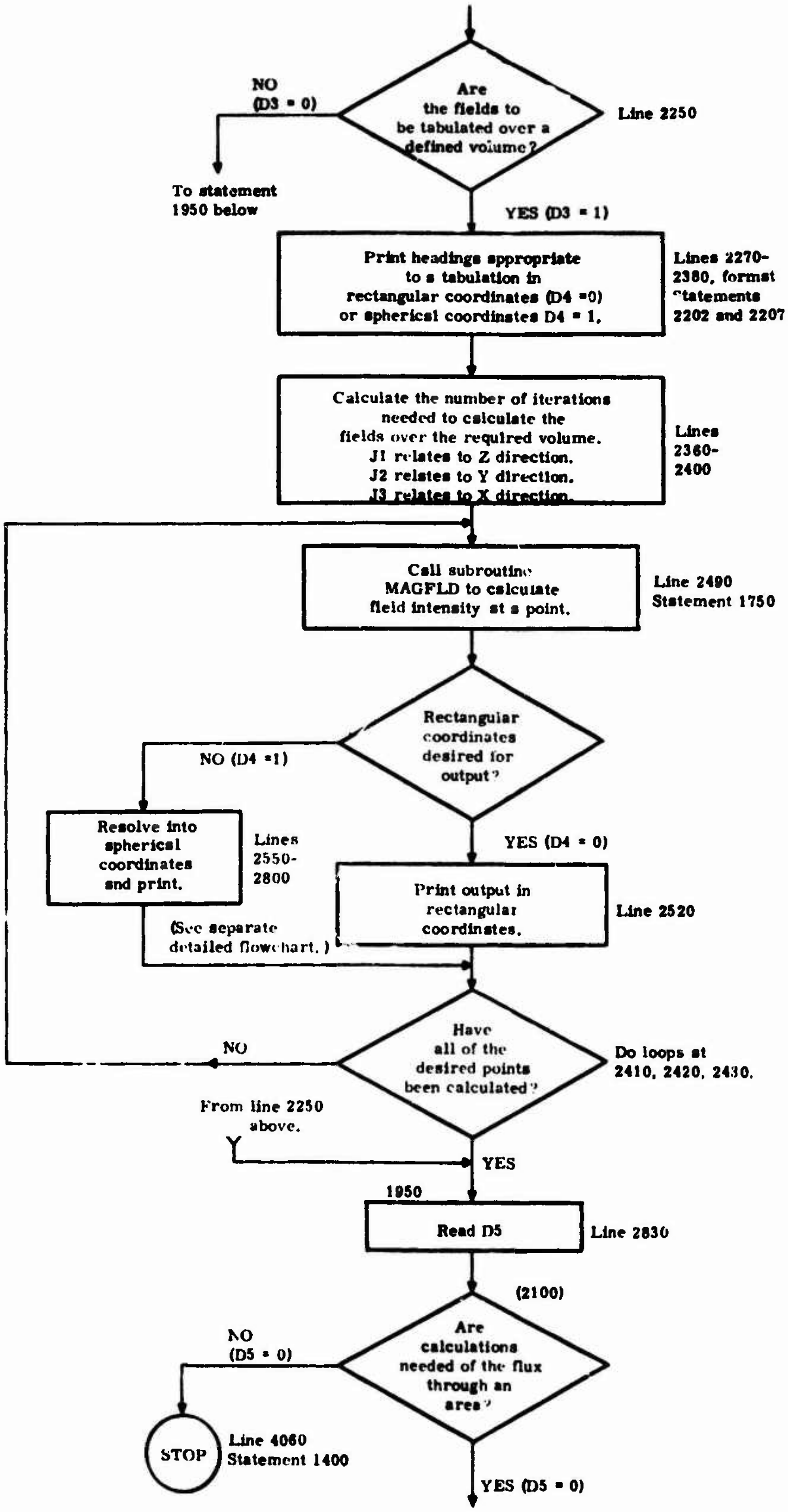


Figure 48. MAIN Program -- Detailed Flowchart (Sheet 2 of 5)

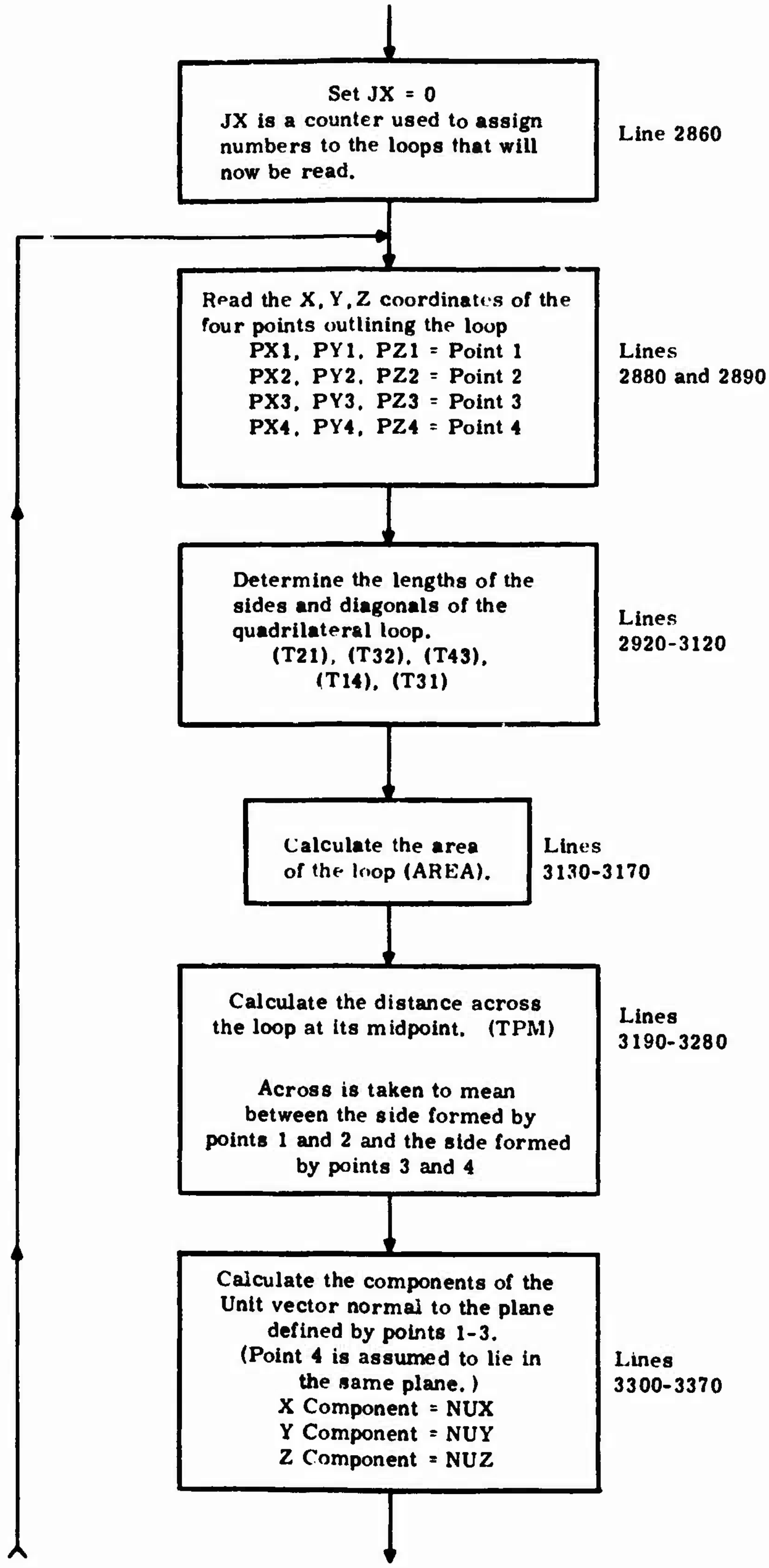


Figure 48. MAIN Program -- Detailed Flowchart (Sheet 3 of 5)

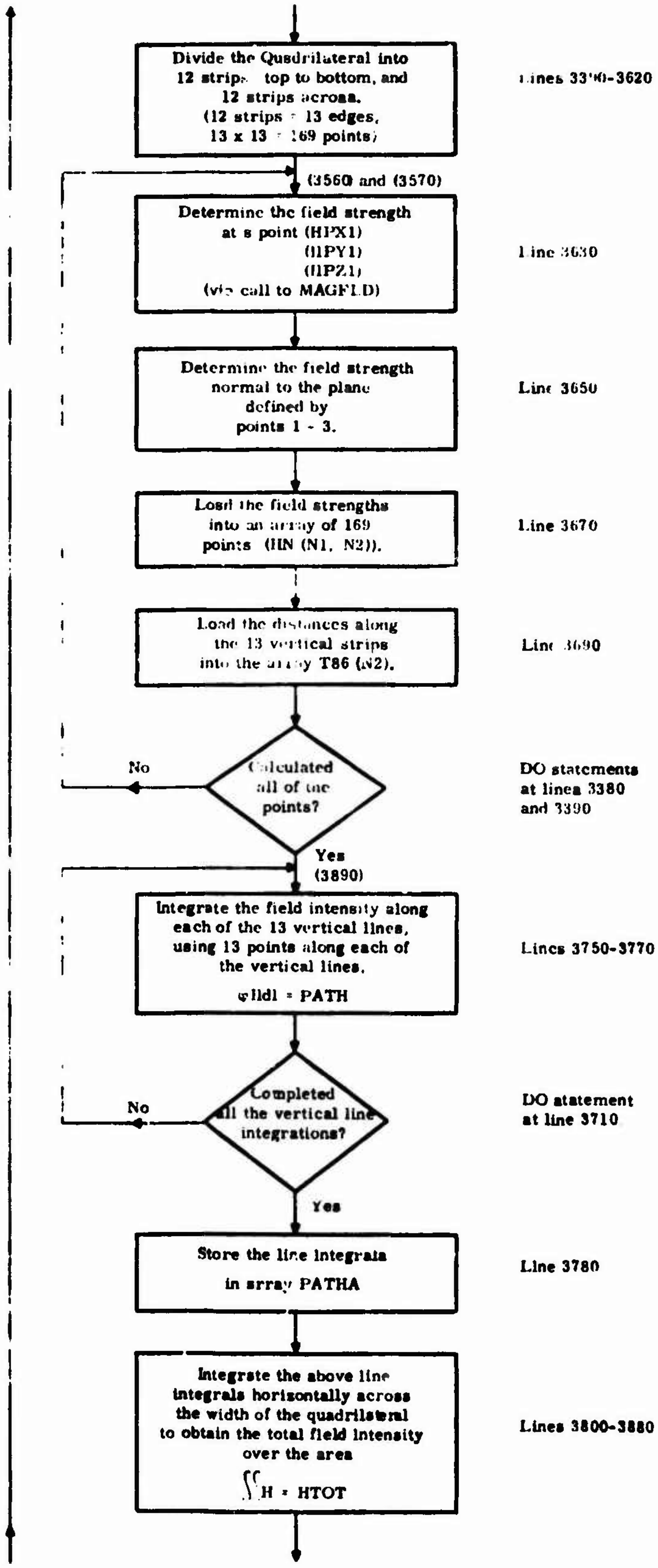


Figure 48. MAIN Program -- Detailed Flowchart (Sheet 4 of 5)

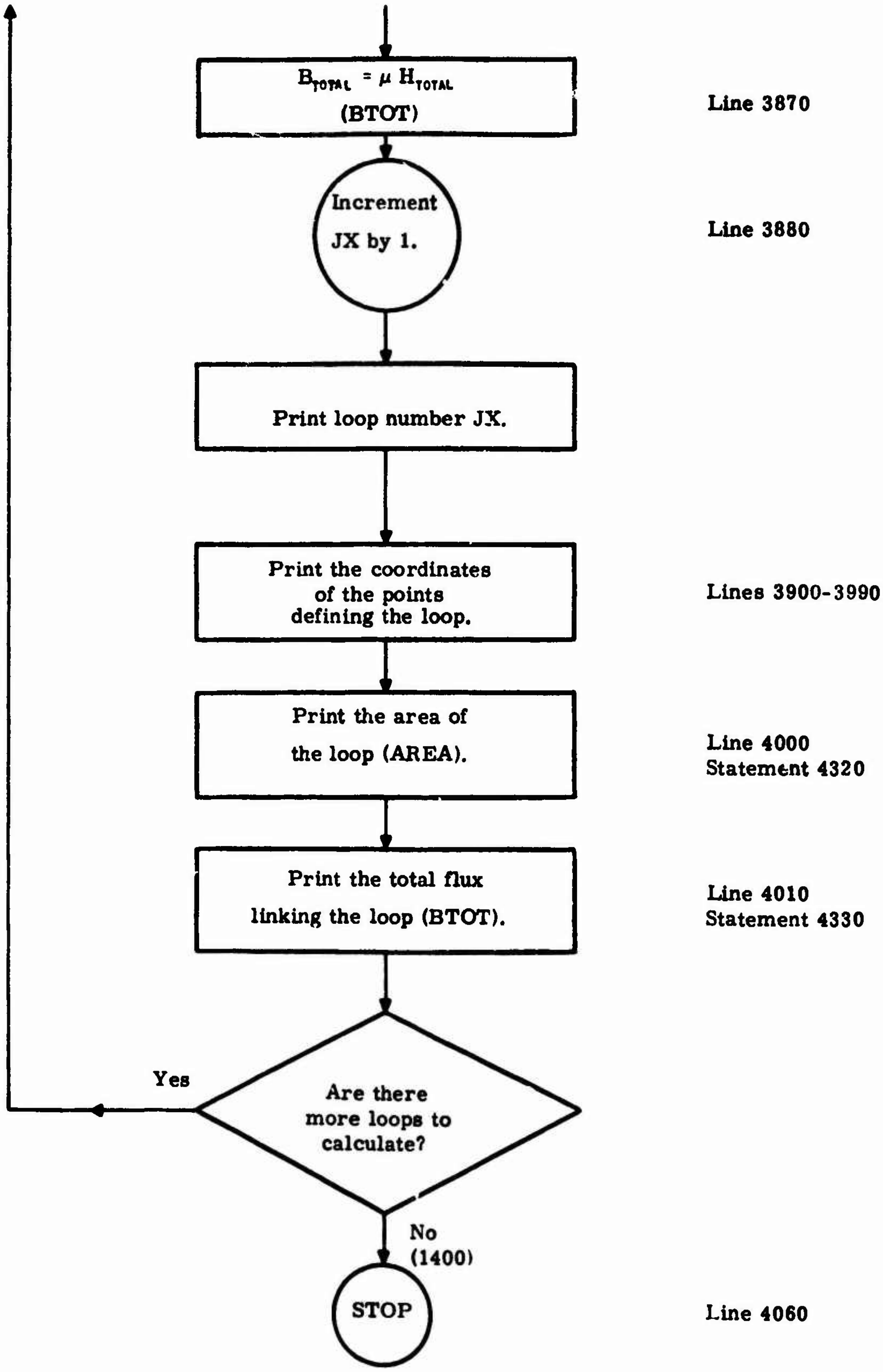


Figure 48. MAIN Program -- Detailed Flowchart (Sheet 5 of 5)

ting surface along the axis. If a reflecting surface is not to be considered, a dummy value is read at this point. Depending on the value of D1 the appropriate heading is then printed. The next quantities read are D3 and D4. D3 is the flag used to indicate whether or not a tabulation is desired of the fields behind the aperture. D4 describes whether the tabulation of field intensities are to be printed in rectangular or spherical coordinates. A dummy value must be entered even if the fields are not to be tabulated. Next read are nine quantities defining the volume over which the magnetic fields are to be tabulated. These are used to set up the appropriate DO loops.

ZPA defines the point along the Z axis at which the tabulation is to start, and ZPB defines the point at which the tabulation is to stop. ZPC defines the interval. YPA, YPB...... ZBC define similar quantities of the X and Y axis.

The effective dipole moments depend upon the size of the aperture. The quantities which control these dipole moments, A11 and A22, are calculated using the subroutine SHAPEFAC. This subroutine is a straightforward evaluation of the equations given earlier in this section under "Theory," and so is not further described by flow charts. The quantity, A33 which is also evaluated by SHAPEFAC, is not used in this program. It relates to the effective electric dipole moment if there were an electric field impinging on the aperture. While the computation routines and housekeeping routines to be described later would evaluate the effects of an electric field, they have not been incorporated in this program at this time.

Lines 2250 through 2400 relate to housekeeping and are self-explanatory in Figure 48. At the end of this housekeeping, there will have been generated a set of coordinates of the point at which the magnetic field is desired. This magnetic field is calculated with the subroutine MAGFLD, which is described below. The magnetic field strengths returned by MAGFLD are then printed in either rectangular or spherical coordinates as requested by the input data. The process by which rectangular coordinates are resolved into spherical coordinates is given on a separate detailed flow chart.

When the above calculations have been finished, the quantity D5 is read. This quantity is a flag used to indicate whether or not calculations are required of the flux through a defined loop. If these calculations are not required, the program stops. If they are required, a counter, JX, is set to zero and the XYZ coordinates of the 4 points outlining the desired loop are read. These steps occupy lines 2860 through 2890.

In lines 2920 through 3170 are calculated the lengths of the sides and diagonals of the quadrilateral loop, and from them the area of the loop.

In lines 3190 to 3280, the distance horizontally across the loop at its midpoint is calculated. "Horizontal" is here taken to be the direction from point P2 to Point P3 or from point P1 to point P4. The term "vertically" is taken

to be in the direction from point P1 to point P2 or point P4 to point P3. These terms in this sense, have no relation to whether the loop itself is oriented horizontally or vertically with respect to the reference XYZ axes.

In lines 3300 through 3370 are calculated the components of the unit vector perpendicular to the plane defined by the loop under consideration. Mathematically this operation consists of taking two vectors that lie in the planes point 1 - point 2, and point 2 - point 3, and taking the cross-product of these two vectors.

Next, the quadrilateral is divided into twelve strips vertically and twelve strips horizontally, and the field strength calculated at the intersection of each of the dividing lines. This makes a total of 169 points. This field strength is calculated by a call to the subroutine MAGFLD. MAGFLD returns the X, Y, Z components of field strength with reference to the original reference axes.

In line 3650, a dot product of the field strength vector and the unit vector normal to the plane is performed in order to determine the component of the magnetic field perpendicular to the loop under consideration. These field strengths are loaded into an array, HN, at line 3670. The vertical distances along each of the thirteen strips are loaded into an array T86 at line 3690.

In lines 3750 through 3770, line integrals of the magnetic field strength are taken along each of the thirteen vertical paths. This integral is evaluated numerically by dividing the vertical strip into two sections of seven points each. One point is common to each of the two sections. The integration routine used is Weddle's rule, which was described under "Theory." These line integrals are stored in the array PATHA. The thirteen line integrals are then integrated horizontally to obtain the total magnetic field linking the plane. The double integral of H is taken in line 3860 and then multiplied by the permeability of air to obtain the total magnetic flux in webers. This latter multiplication is taken at line 3870.

Finally, the loop number, JX, the coordinates of the points defining the loop, the area of the loop, and the total magnetic flux linking the loop are printed out. The program then loops back to read in the coordinates of additional points, if there are any additional loops to be considered. If no data are found in the input files, the program stops.

<u>Rectangular to Spherical Coordinates</u>

Figure 49 is a detailed flow chart of the process by which the rectangular coordinates of the magnetic field are resolved into spherical coordinates.

Figure 50 shows the conventions regarding the designation of the spherical coordinates. There are two angles to be calculated: the latitude angle, the

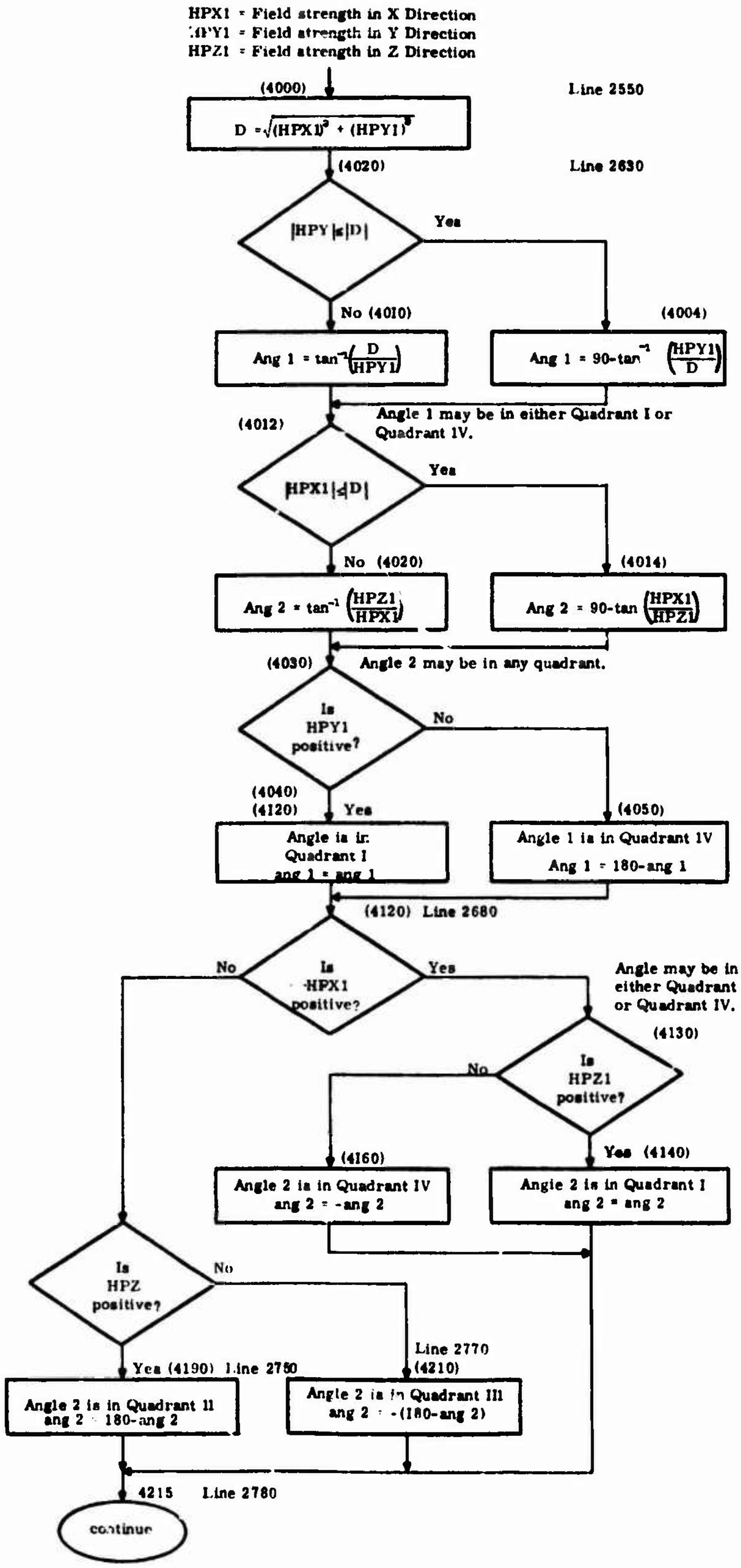


Figure 49. Detailed Flowchart

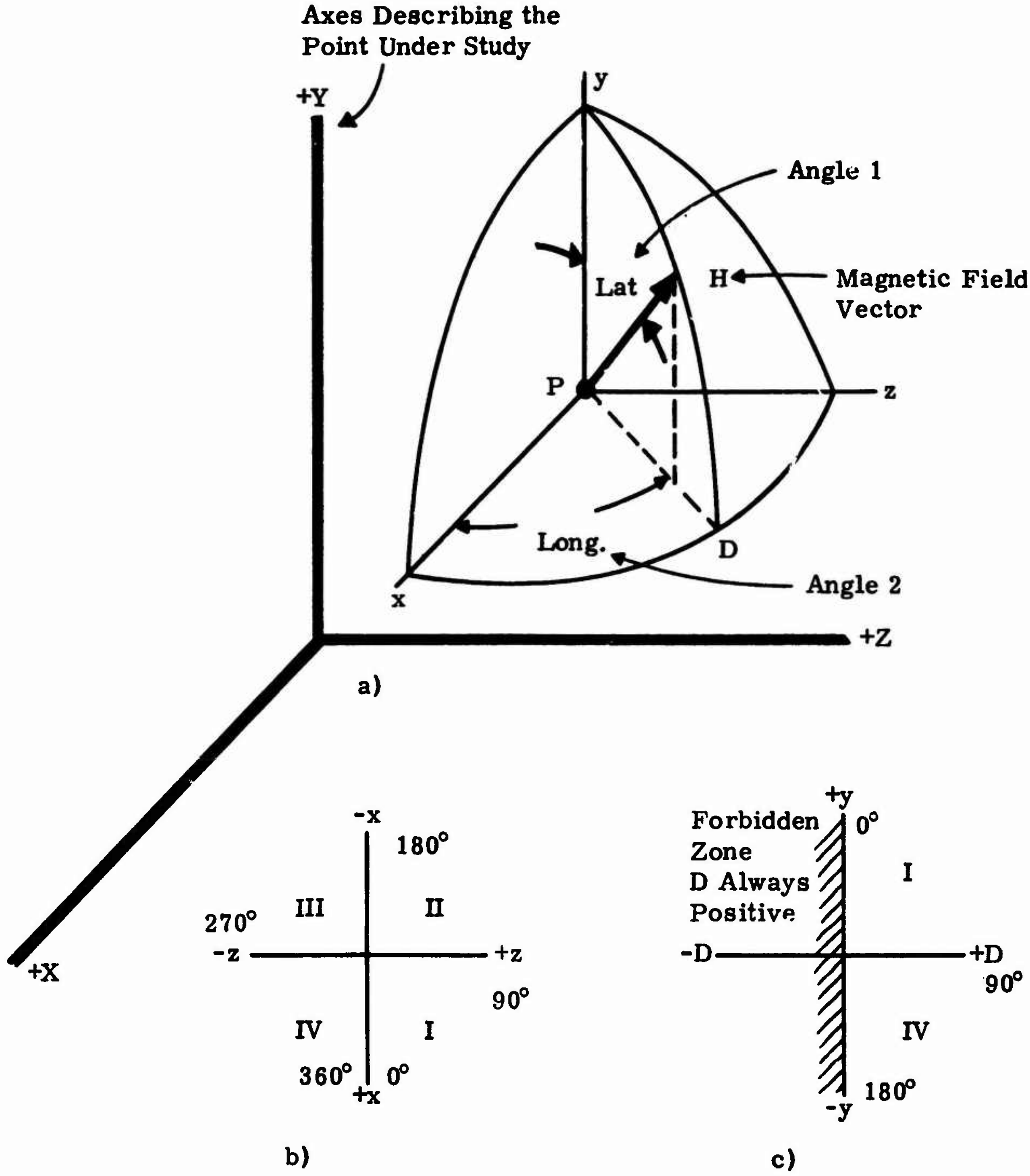


Figure 50. Conventions Regarding Angles: a) Latitude and Longitude Angles Defined; b) Quadrant Designations for Longitude; c) Quadrant Designations for Latitude

angle with respect to the vertical Y axis; and the longitude angle, the angle in the XZ plane from the positive X axis. Lest there be confusion as to why the positive X axis points to the left, remember that Figure 50 shows the region behind the aperture. When viewed from outside, where the magnetic field originates, the reference X axis has its positive sense to the right. The latitude and longitude angles are called ANG1 and ANG2, respectively, in the program.

These angles are basically calculated from the arc tangent of the respective components, HX and HZ for ANG2 (LONG) and HY and HD for ANG1 (LAT). D is the length of the projection of the H field vector in the XZ plane. There are two problems in this resolution. The first is to ensure that under no condition does the denominator in the argument for the arc tangent go to zero. If it does go to zero, appropriate angles are calculated, but annoying error messages are still generated and printed by the computer. This is prevented from occurring by the switch at statement 4020, line 2630, and the alternate methods of calculating the angle at statements 4010 and 4004. The appropriate switch and statements for ANG2 occur at statements 4012, 4014, and 4020.

The second problem relates to determining the appropriate quadrant in which the angle lies, since the arc tangent routine does not intrinsically resolve quadrants. Upon evaluation of the arc tangents, angle 1 may be in either quadrant 1 or 4. Quadrants 2 and 3 are forbidden regions, because the polarity of the D component is always positive, inasmuch as it is taken by the vector addition of the X and Z components. Angle 2 may be in any quadrant. The switches at statements 4030, 4120, 4140, and 4160, resolve the question of appropriate quadrants. Appropriate statements add or subtract 180° or reverse the sign of the angles. The logic is straightforward, though a bit involved, and is shown on the remainder of Figure 49.

MAGFLD Program

The major subroutine used in the program APERATURE is MAGFLD. Figure 51 is the flow chart for this subroutine. The program is entered at line 4320, using the quantities shown at the top of Sheet 1. After the initial quantities are defined, the first task performed is to translate the coordinates of the point under study from the original reference X and Y axes to a new set of axes, oriented along the major and minor axes of the aperture. This is done at lines 4380 through 4430. The Z coordinate of the point under study is also shifted to a new Z axis, centered on the aperture. The distances from the middle of the aperture to the point under study, in terms of the new coordinate geometry, are then calculated in lines 4480 through 4510.

The quantities C1 and C2 are then calculated. C1 and C2 are basically the distances from the center of the aperture to the point under study, although they also include a term related to the length of the major and minor axes of the aperture. Accordingly, these terms cannot go to zero, even if the point under study were to be at the center of the aperture.

Quantities Used to Enter MAGFLD Are:

ANGA - Angle aperture makes to X axis
ANGH - Angle field makes to X axis
XP1, YP1, ZP1 - X, Y, Z coordinates of point at which field is to be calculated.
XA, YA, ZA - X, Y, Z coordinates of center of aperture
HEXT - External magnetic field strength
A11 - Shapefactor for major axis of aperture
A22 - Shapefactor for minor axis of aperture
L1 - Length of major axis of aperture
L2 - Length of minor axis of aperture
D1 - YES/NO as regards reflecting surface
D2 - Z coordinate of reflecting surface

Quantities Returned by MAGFLD Are:

HPX1, HPY1, HPZ1 - Magnetic field strengths in X, Y, Z directions at point under calculation

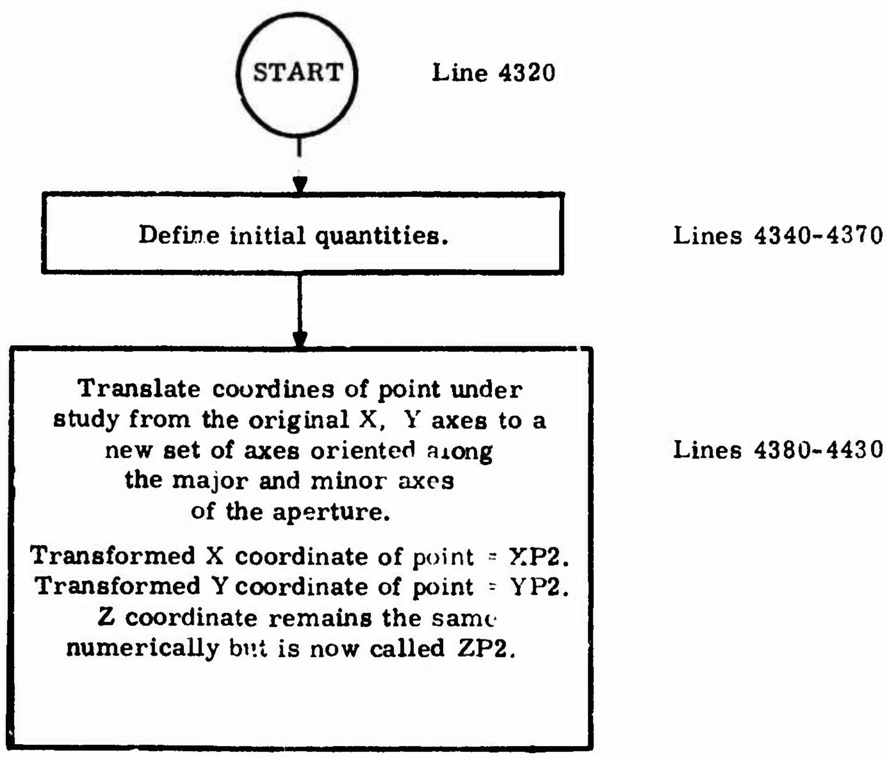


Figure 51. Flow Chart for Subroutine MAGFLD (Sheet 1 of 3)

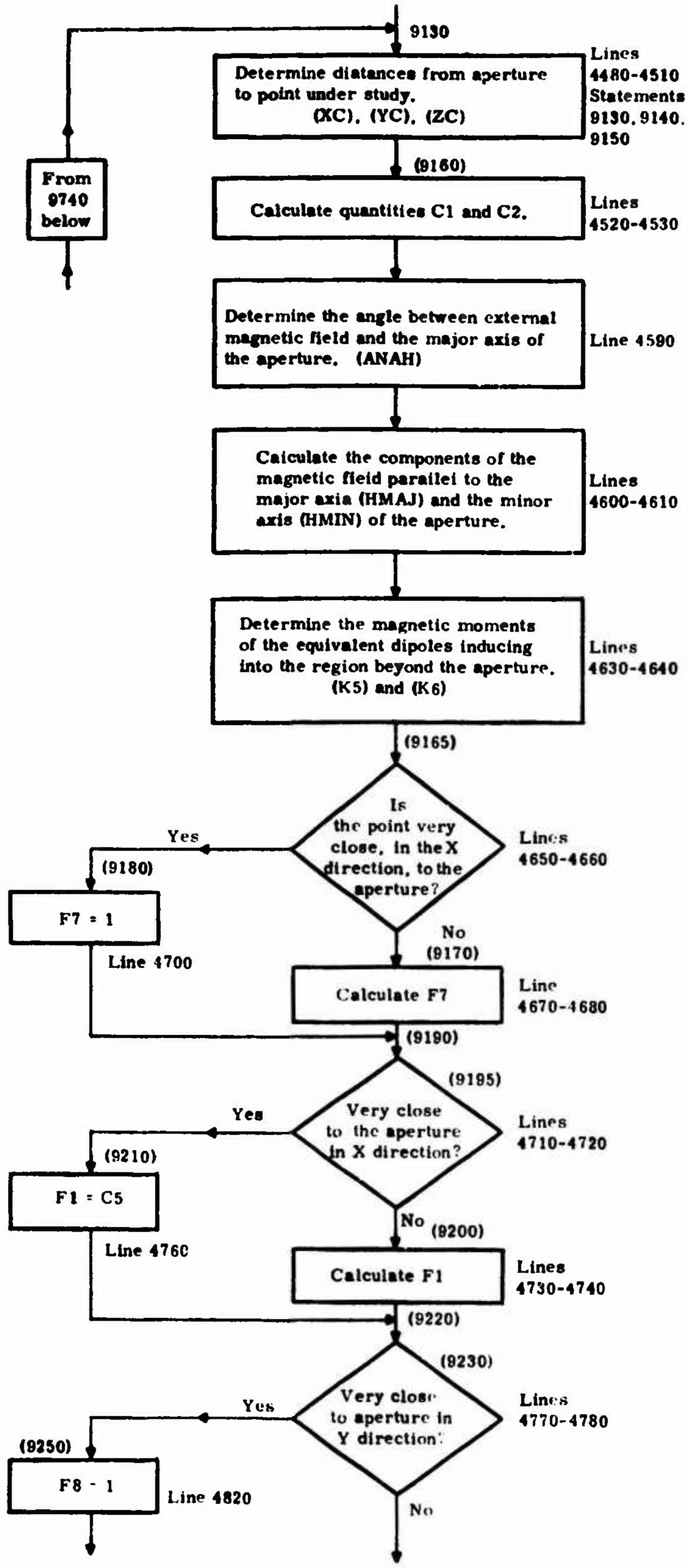


Figure 51. Flow Chart for Subroutine MAGFLD (Sheet 2 of 3)

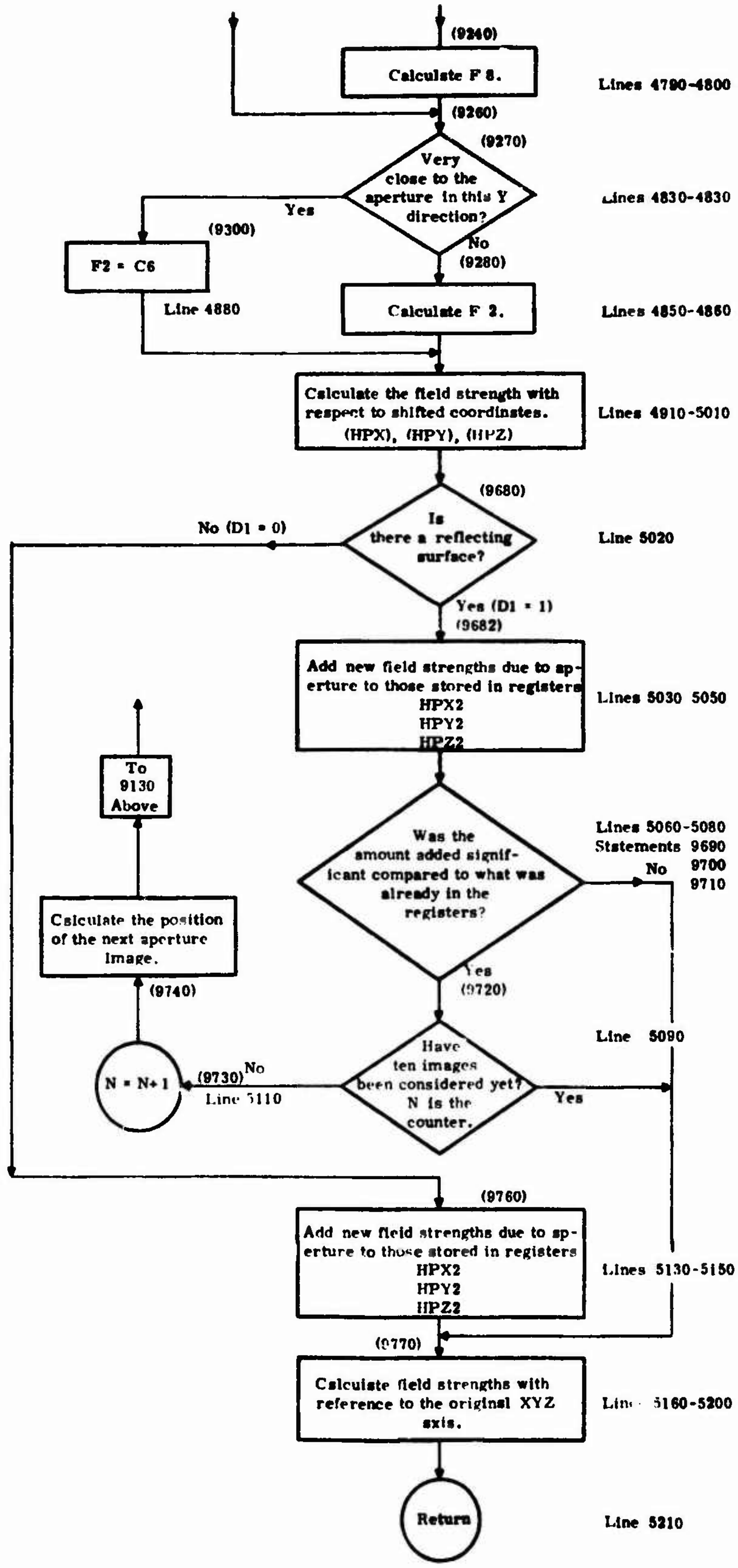


Figure 51. Flow Chart for Subroutine MAGFLD (Sheet 3 of 3)

In subsection "Theory," the field strength at the point under study is seemed to be that due to the magnetic moment of the dipoles formed by the major and the minor axes of the elliptical aperture. These dipole moments are the product of the magnetic field strength along the major and minor axes of the aperture, the lengths of the major and minor axes, and the shape factors for the aperture. These shape factors, which also include the lengths of the major and minor axes, were calculated in the subroutine SHAPEFAC. These factors are all calculated in lines 4580 through 4640.

In subsection "Theory," the field strength equations were presented in terms of the subfactors F1 through F4 and G1 through G8. In the subroutine, the quantities F1 and F2 are the same as the quantities F1 and F2 derived in the subsection on theory; quantities F3 and F4 mentioned in there are, however, replaced by their corresponding equivalents, F7 and F8. The quantities F3 through F6 in the subroutine are not related to any corresponding quantities in the subsection on theory.

The quantities F1, F2, F7 and F8 involve raising the quantities C1 and C2 to powers up to and including the 7th power. When the point under study is very close to the X and Y axes, but not exactly on the axes, underflow conditions are generated in the computer. Correct numerical answers are returned, but annoying error messages are still printed. In order to eliminate these error messages, there are switches at statements 9165, 9195, 9230, and 9270 which, when appropriate, calculate the quantities F1, F2, F7, and F8 by their small argument equivalents. This process of evaluating the terms in the field equations occupies the space from lines 4650 through 4860. The components of the magnetic field strengths are then evaluated at lines 4990 through 5010.

At this point the presence or absence of a reflecting surface must be treated. If a reflecting surface is not present, as indicated by the switch at line 5020 or in statement 9680, the calculated magnetic field vectors are rotated back to the original reference axis in lines 5180 through 5200 and the quantities HPX1, HPY1, and HPZ1 are returned to the program MAIN.

If a reflecting surface is present, the field components calculated are added to the contents of the storage registers HPX2, HPY2, and HPZ2. (Initially these storage registers had been set to zero at lines 4440 through 4460.) The program then determines whether the quantities added to the storage registers HPX2, HPY2, and HPZ2 were significant compared to what was already in the registers. For this first loop through the program the quantities of course were significant; there was nothing stored in those registers to begin with. The program then loops back to calculate the field strengths produced by the first reflection of the aperture in the reflecting surface. The position of the reflection is along the Z axis at a spacing from the original aperture equal to twice the spacing to the reflecting surface. This new position along the Z axis is calculated at line 5110, statement 9740.

The program then adds the field strengths produced by successive reflections to those stored in the registers HPX2, HPY2, and HPZ2, testing at each time to see whether the contribution from the aperture under study was significant enough to bother with. The number of times through this loop is counted with the counter N. The coordinate of the aperture under study increases rapidly as the program goes through this cycle, and eventually the contribution to the total field strength from the higher order reflections becomes negligible. The counter N is used at the switch point 9720 to break out of the loop if the field strength has not converged to its final value after treating ten images. If the contribution from the last image was negligible, or if ten images had been considered, the field strengths in the registers are then rotated back to the original X, Y, Z axes, and the quantities HPX1, HPY1, and HPZ1 are returned to the program at MAIN.

To ease the task of going through the program, Table 1 lists the statement numbers in ascending order versus their corresponding line numbers. If the program were to be sequenced the line numbers would change.

VALIDATION OF APERTURE

Validation of the computer program APERTURE is based upon a comparison of the computer-generated results with the results predicted by classical electromagnetic theory -- that the flux density due to a magnetic dipole decreases as a function of $1/r^3$ for large values of r. The computer results are shown in Figures 52 through 54.

These figures show the computer results to be in agreement with this $1/r^3$ decrease. The orientation of the vector field is also equivalent to that predicted.

Table 1

STATEMENT NUMBERS VERSUS LINE NUMBERS

Line No.	Statement No.	Line No.	Statement No.
10	1660	4000	2550
15	1670	4002	2560
20	1660	4004	2570
30	1690	4006	2580
110	1710	4010	2590
115	1720	4012	2600
120	1730	4014	2610
122	1740	4016	2620
123	1750	4020	2830
130	1770	4030	2640
140	1790	4040	2650
145	1800	4050	2660
150	1610	4110	2670
155	1820	4120	2660
160	1630	4130	2690
165	1640	4140	2700
170	1850	4150	2710
175	1860	4160	2720
180	1670	4170	2730
185	1860	4180	2740
186	1900	4190	2750
190	1910	4200	2760
192	1920	4210	2770
195	1930	4215	2760
200	1940	4230	2600
210	1950	4315	3890
220	1960	4320	4000
275	2110	4330	4010
260	2120	4340	4020
265	2140	4380	4040
290	2150	5230	5220
295	2170	9130	4490
1002	2470	9140	4500
1199	2460	9150	4510
1210	2520	9160	4520
1400	4060	9165	4860
1750	2490	9170	4670
1950	2610	9160	4700
1955	2650	9190	4710
1957	2670	9195	4720
1960	4050	9200	4730
2100	2840	9210	4760
2200	2280	9220	4770
2201	2270	9230	4780
2202	2280	9240	4790
2204	2360	9250	4820
2206	2310	9260	4830
2207	2320	9270	4640
2206	2340	9260	4650
2450	2370	9300	4680
3130	4280	9320	4900
3140	4270	9660	5020
3160	4290	9862	5030
3170	4300	9890	5060
3370	2510	9700	5070
3550	3560	9710	5060
3560	3390	9720	5090
3570	3400	9730	5100
3860	3700	9740	5110
3690	3710	9760	5130
3950	3790	9770	5155

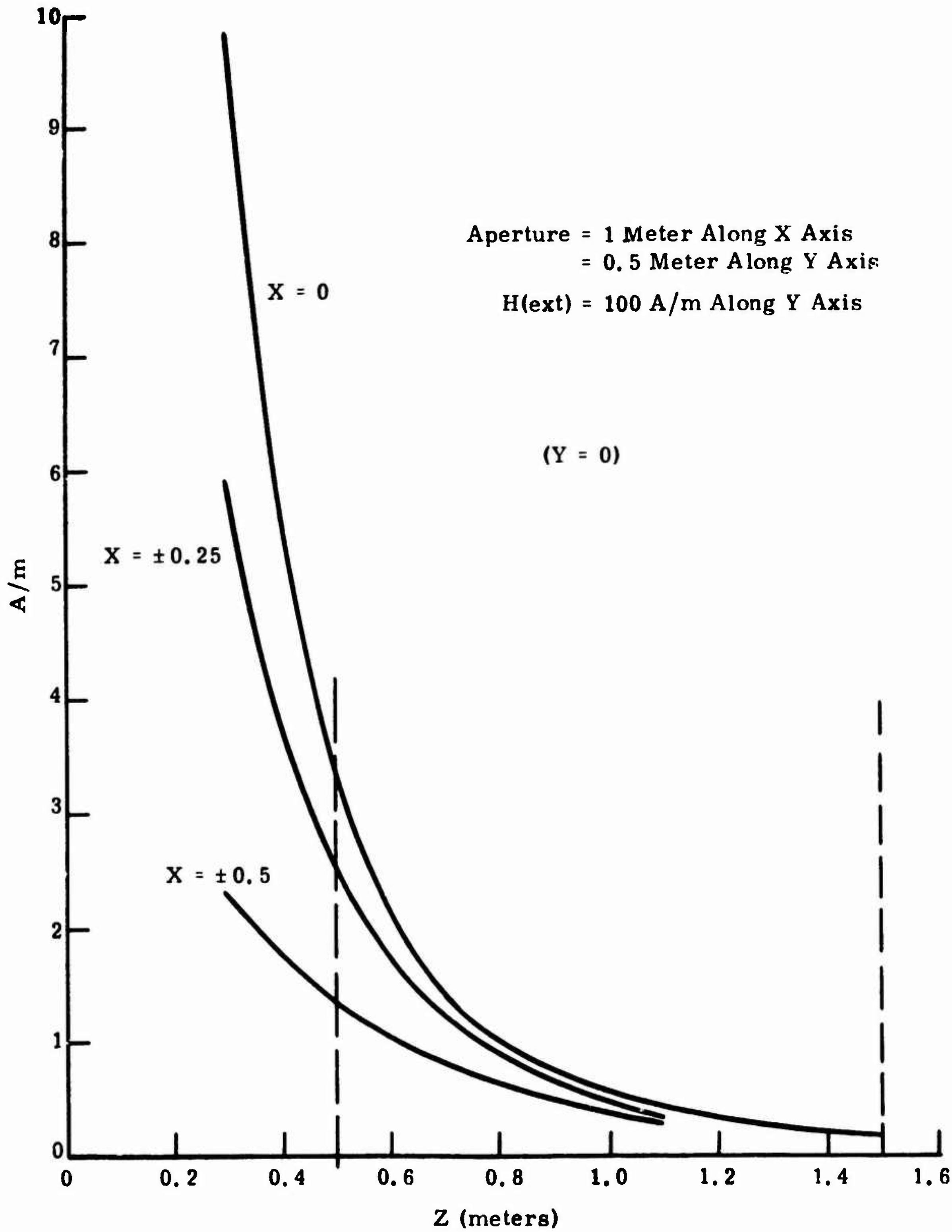


Figure 52. Field Intensity - Y Component

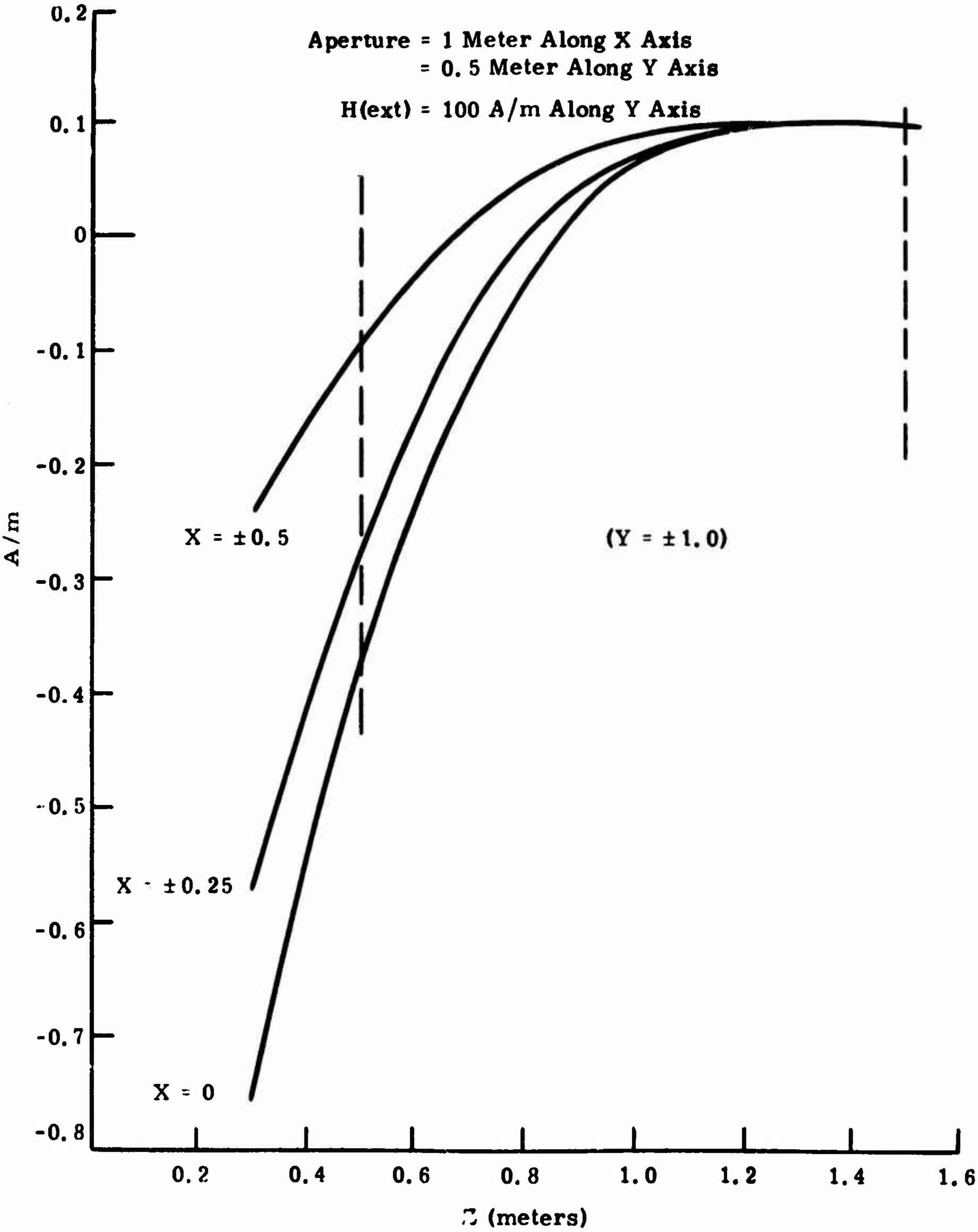


Figure 53. Field Intensity - Y Component

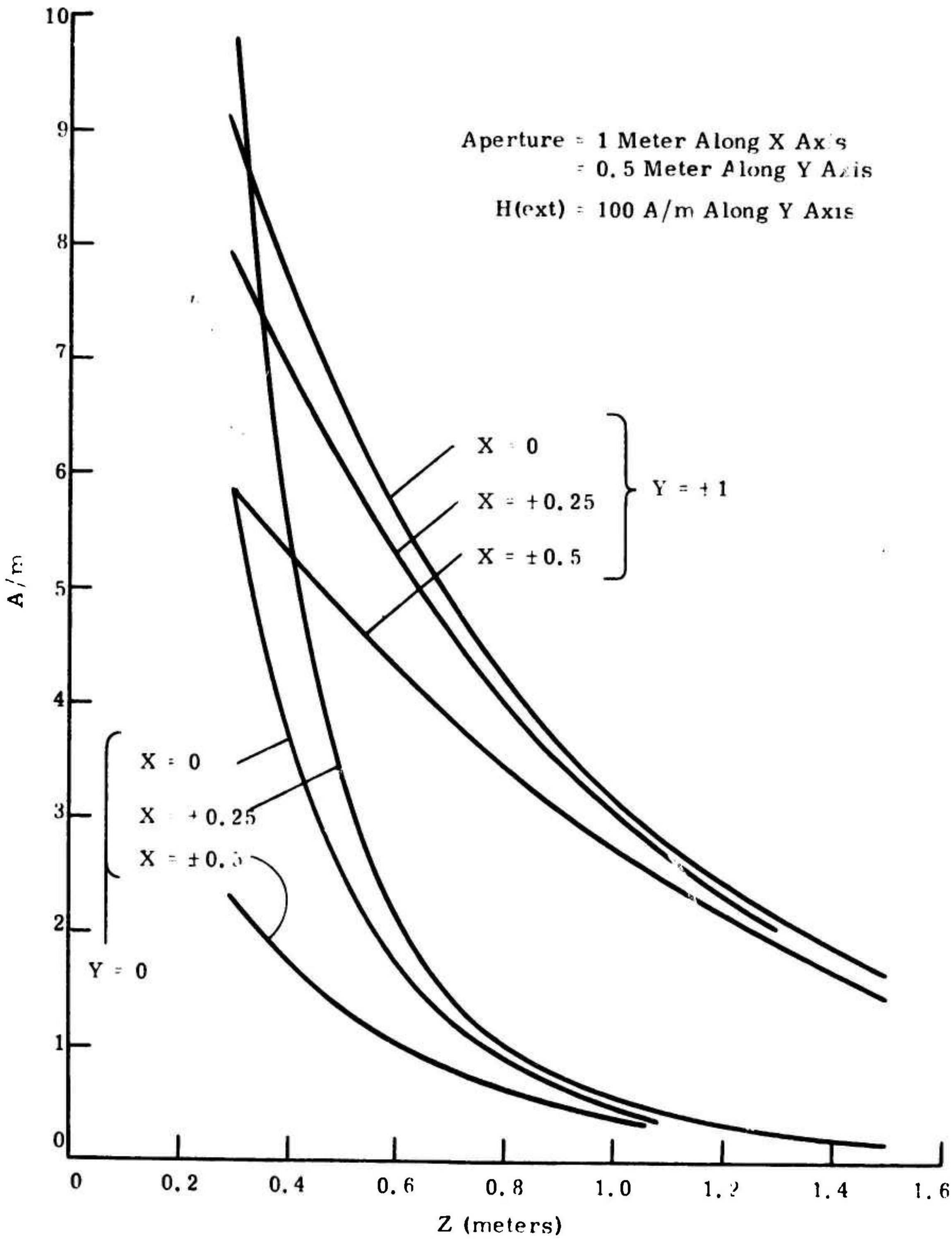


Figure 54. Total Field Intensity

Section 4

CONCLUSIONS

Two computer programs, APERTURE and DIFFUSION, have been developed for the calculation of probable electromagnetic fields and resulting induced voltages in aircraft electrical circuits. These programs should enable the researcher or system designer to determine the order of magnitude of lightning induced voltages which may be induced in simple circuit geometries by lightning strokes of any assumed amplitude and waveform. By variation of the conductor location parameters, the programs enable the designer to determine the best location (i.e., where coupling is minimized) within the airframe for placement of conductors.

The DIFFUSION program is based on calculation of the magnetic fields which occur inside an airframe as a result of lightning current diffusing to the inside surface of its metallic skin. The program therefore assumes that the airframe skin is metallic and has no apertures. This is the flux which normally exists inside an all-metallic airframe, and should be considered as the minimum to which internal fields can be reduced in a metallic airframe of given skin material and thickness by such means as closure of apertures and improvements in electrical bonding.

Because diffusion fields are of relatively low amplitude and slower rates of rise than their external counterparts, voltages induced by diffusion fields linking small circuit loop areas such as those formed between parallel pair or twisted pair conductors are likely to be small. On the other hand, large loops, such as those formed between either conductor of a pair and the airframe, may receive high induced voltages from diffusion fields. This is especially true because the diffusion fields are usually present throughout the entire length of such a circuit.

The APERTURE program calculates the fields penetrating the interior of the airframe from a given field tangential (in any assumed direction) to the outside surface of the airframe at the aperture in question. These fields penetrate directly into the interior of the airframe but are strong only in the vicinity of the aperture. If a parallel pair of conductors passes nearby, the aperture fields are often of great enough amplitude and rate of rise to induce large voltages. If located some distance away from the aperture, however, resulting induced voltages may be small, because the field intensity falls off as the square or cube of the distance from the aperture.

Thus a complete analysis of a particular situation will usually require the use of both computer programs and superposition of the results of one on those of the other for consideration of the worse case.

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At present, APERTURE and DIFFUSION deal with relatively basic geometries and do not account for such details as internal structural components (e.g., spars and ribs), concentration of lightning current around the points of stroke entry, or leakage through resistive joints or bonds. It will therefore be desirable to develop further refinements to permit consideration and accurate calculation of the effects of such details as ribs, spars, seams, access doors, flap openings, as well as such other objects as antennas and radomes. Each of these additions should be validated by comparison with measured test data obtained from other programs of aircraft lightning induced voltage measurement.

It may also be advantageous to convert the input and output formats of the programs to the same format as the one used in the Air Force intersystem analysis program (IAP). The latter is a frequency-domain input/output format which expresses intersystem electromagnetic interference (EMI) in terms of its frequency spectral content (energy at each frequency within a wide bandwidth of frequencies). Basically, conversion of the basic lightning induced voltage model to this format will require conversions of the calculated induced impulse voltages to their Fourier spectral coefficient equivalents; the interference from lightning is therefore expressed in the same frequency spectral language as the EMI already calculated by the IAP. Other format changes will involve airframe geometrical descriptions. These changes are not expected to be extensive, however, and if made may promote use of these lightning induced voltage models by engineers concerned with the solution of related EMI problems as well.

Section 5

REFERENCES

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5. J.A. Plumer, Analysis and Calculation of Lightning Induced Voltages in Aircraft Electric Circuits, Final Report No. CR-2349;SRD-72-066, National Aeronautics and Space Administration Contract No. NAS3-14836, General Electric Company, Pittsfield, Mass., January 1974.

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8. R.S. Burington, Handbook of Mathematical Tables and Formulas, Handbook Publishers Inc., Sandusky, Ohio, 1958.

9. F.A. Fisher, General Electric Corporate Research and Development, unpublished work, 1974.

10. C.D. Taylor, "Electromagnetic Pulse Penetration through Small Apertures," Interaction Note 74, Electromagnetic Pulse Interaction Notes, Vol. 5, Air Force Weapons Laboratory, Kirtland Air Force Base, N.M., March 1973.

11. J.B. Scarborough, Numerical Mathematical Analysis, 4th ed., Johns Hopkins Press, Baltimore, Md., 1958, p. 133.

Appendix I

DERIVATION OF TOTAL FLUX

The purpose of the derivation given here is to find the total flux, ψ, generated by the current from a given filament, passing through an area bounded by lines parallel to the wire at distances D_1 and D_2 from it, along the wire from ℓ_2 to ℓ_1. Points ℓ_1 and ℓ_2 (shown in Figure 55) are the beginning and end of a circuit conductor.

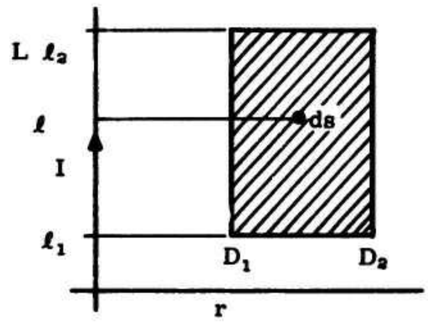

Figure 55. Geometry of Flux Derivation

$$\text{The total flux is } \psi = \int_{D_1}^{D_2} \int_{\ell_1}^{\ell_2} B \cdot ds \tag{121}$$

$$B(\ell\, r) = \frac{\mu_0 I}{4\pi}\left[\frac{\ell}{r\sqrt{\ell^2 + r^2}} + \frac{L-\ell}{r\sqrt{(L-\ell)^2 + r^2}}\right] \tag{122}$$

$$\psi = \frac{\mu_0 I}{4\pi} \int_{D_1}^{D_2} \int_{\ell_1}^{\ell_2} \frac{\ell}{r\sqrt{\ell^2 + r^2}} + \frac{L-\ell}{r\sqrt{(L-\ell)^2 + r^2}}\, d\ell\, dr \tag{123}$$

This comprises three separate integrals:

$$\psi = \frac{\mu_0 I}{4\pi} \int_{D_1}^{D_2} \int_{\ell_1}^{\ell_2} \frac{\ell}{r\sqrt{\ell^2 + r^2}}\, d\ell dr + \frac{\mu_0 I}{4\pi} \int_{D_1}^{D_2} \int_{\ell_1}^{\ell_2} \frac{L}{r\sqrt{(L-\ell)^2 + r^2}}\, d\ell\, dr$$

$$- \frac{\mu_0 I}{4\pi} \int_{D_1}^{D_2} \int_{\ell_1}^{\ell_2} \frac{\ell}{r\sqrt{(L-\ell)^2 + r^2}}\, d\ell\, dr \tag{124}$$

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$$\psi = \frac{\mu_0 I}{4\pi} \int_{D_1}^{D_2} \left[\frac{\sqrt{\ell^2 + r^2}}{r} \right]_{\ell_1}^{\ell_2} dr$$

$$+ \frac{\mu_0 I}{4\pi} \int_{D_1}^{D_2} \left[\frac{L}{r} (\log (2\ell - 2L + 2 \sqrt{\ell^2 - 2L\ell + L^2 + r^2})) \right]_{\ell_1}^{\ell_2} dr$$

$$- \frac{\mu_0 I}{4\pi} \int_{D_1}^{D_2} \left[\frac{1}{r} (\sqrt{\ell^2 - 2L\ell + L^2 + r^2} + L \log (2\ell - 2L \qquad (125)$$

$$+ 2 \sqrt{\ell^2 - 2L\ell + L^2 + r^2} \right]_{\ell_1}^{\ell_2} dr$$

Rearranging terms and substituting ℓ_1 and ℓ_2,

$$\psi = \frac{\mu_0 I}{4\pi} \int_{D_1}^{D_2} \left[\frac{\sqrt{\ell_2^2 + r^2}}{r} - \frac{\sqrt{\ell_1^2 + r^2}}{r} \right.$$

$$+ \frac{\sqrt{\ell_1^2 - 2L\ell_1 + L^2 + r_2}}{r} - \frac{\sqrt{\ell_2^2 - 2L\ell_2 + L^2 + r^2}}{r}$$

$$+ \frac{L}{r} \log (2\ell_2 - 2L + 2 \sqrt{\ell_2^2 - 2L\ell_2 + L^2 + r^2})$$

$$- \frac{L}{r} \log (2\ell_2 - 2L + 2 \sqrt{\ell_2^2 - 2L\ell_2 + L^2 + r^2}$$

$$+ \frac{L}{r} \log (2\ell_1 - 2L + 2 \sqrt{\ell_1^2 - 2L\ell_1 + L^2 + r^2})$$

$$\left. - \frac{L}{r} \log (2\ell_1 - 2L + 2 \sqrt{\ell_1^2 - 2L\ell_1 + L^2 + r^2}) \right] dr \qquad (126)$$

$$\psi = \frac{\mu_0 I}{4\pi} \int_{D_1}^{D_2} \left[\frac{\sqrt{\ell_2^2 + r^2}}{r} - \frac{\sqrt{\ell_1^2 - r^2}}{r} + \frac{\sqrt{(\ell_1 - L) + r^2}}{r} - \frac{\sqrt{(\ell_2 - L)^2 + r^2}}{r} \right] dr$$

$$(127)$$

$$\psi = \frac{\mu_0 I}{4\pi}\left[\sqrt{\ell_2^2 + r^2} + \ell_2 \log\left(\frac{\sqrt{\ell_2^2 + r^2} - \ell_2}{r}\right)\right.$$

$$- \sqrt{\ell_1^2 + r^2} - \ell_1 \log\left(\frac{\sqrt{\ell_1^2 + r^2} - \ell_1}{r}\right)$$

$$+ \sqrt{(\ell_1 - L)^2 + r^2} - (\ell_1 - L)\log\left(\frac{\ell_1 - L + \sqrt{(\ell_1 - L)^2 + r^2}}{r}\right)$$

$$\left. - \sqrt{(\ell_2 - L)^2 + r^2} + (\ell_2 - L)\log\left(\frac{\ell_2 - L + \sqrt{(\ell_2 - L)^2 + r^2}}{r}\right)\right]_{r = D_1}^{r = D_2} \quad (128)$$

Appendix II

FUSELAGE

The geometry of the fuselage model is broken down to straight line segments and circular sections. These configurations can be used to describe the front view of most fuselage geometries. To use the program the fuselage is laid out as in Figure 56. The left side and bottom are along the XY axis. The top is at $Y = Y_1$ and the right side at $X = X_1$. The circular sections in the corners are set up so that the radii of the top sections are the same and the lower sections are the same.

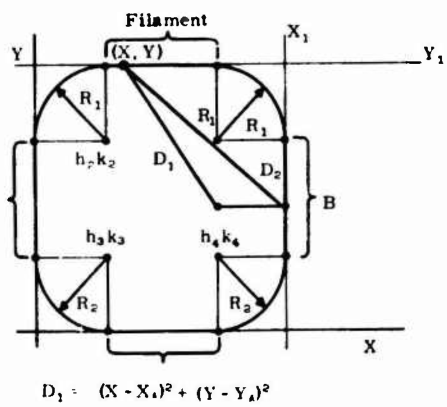

Figure 56. Fuselage Front View

It is anticipated that typical aircraft will be symmetrical from side to side on the top and on the bottom, but that the lower section may differ from the upper section in curvature. This method can be used directly on a structure that has no straight segments, representing the minimum program to encompass the maximum geometry anticipated.

If the fuselage is tapered, the program, which assumes no taper, can be sectioned into two or more straight pieces; the same setup would be used, with different physical sizes.

The front view of a typical fuselage is shown in Figure 56. The distances D_1 and D_2 are calculated for the filaments along the straight line sections and the four curved sections.

In the straight sections of Figure 56,

$$D_1 = \sqrt{(X - X_A)^2 + (Y - Y_A)^2} \tag{129}$$

- For the section from 0, k_3 to 0, k_2:

  X = 0, step Y in J steps from k_3 to k_2

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- For the section from h_2, Y_1 to $h_1 Y_1$:

  $Y=Y_1$, step X in J steps from k_2 to h_1
- For the section from X_1, k_1 to $X_1 k_4$:

  $X = X_1$, step Y in J steps from k_1 to k_4
- For the section from h_4, 0 to h_3 0:

  $Y=0$, step X in J steps from h_3 to h_4

For the curved sections use Figure 57.

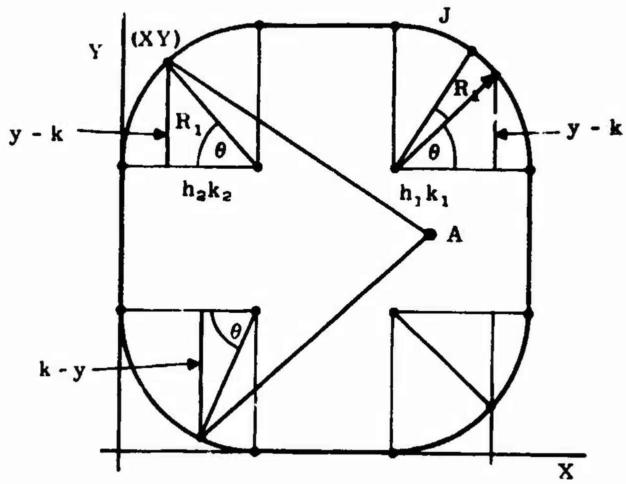

Figure 57. Curved Sections Geometry

For the first and second quadrants,

$$k_1 = k_2 = k$$

$$\sin\theta = \frac{y-k}{R_1}$$

$$y = R_1 \sin\theta + k$$

The equation for each section is:

$$(X-h)^2 + (Y-k)^2 = R_1^2 \tag{130}$$

with h and k as h_1k_1 for the first quadrant and k_2h_2 for the second quadrant:

$$(X - h)^2 = R_1^2 - (Y - k)^2 \tag{131}$$

$$X = \sqrt{R_1^2 - (Y-k)^2} + h \tag{132}$$

Substitute $Y = R_1 \sin\theta + k$:

$$X = \sqrt{R_1^2 - (R_1 \sin\theta + k - k)^2} + h = \sqrt{R_1^2 - (R_1 \sin\theta)^2} + h \tag{133}$$

$$D_1 = \sqrt{(X - X_A)^2 + (Y - Y_A)^2} \tag{134}$$

Substitute for X and Y

$$D_1 = \sqrt{\left(\sqrt{R_1^2 - (R_1 \sin\theta)^2} + h - X_A\right)^2 + (R_1 \sin\theta + k - Y_A)^2} \qquad (135)$$

D_2 is D_1 with $X_B Y_B$ substituted for $X_A Y_A$:

$$D_1 = \sqrt{(R_1 \cos\theta + h - X_A)^2 + (R_1 \sin\theta = k - Y_A)^2} \qquad (136)$$

First quadrant: step θ from 0 to $\frac{\pi}{2}$ in $\frac{J}{R_1}$ steps.

Second quadrant: step θ from $\frac{\pi}{2}$ to π in $\frac{J}{R_1}$ increments.

For the third and fourth quadrants:

$$\sin\theta = \frac{k-Y}{R} \qquad Y = k - R\sin\theta$$

X is still the same, with R_2 substituted for R_1:

$$X = \sqrt{R_2^2 - (R_2 \sin\theta)^2} + h \qquad (137)$$

$$D_1 = \sqrt{(X - X_A)^2 + (Y - Y_A)^2} \qquad (138)$$

$$D_1 = \sqrt{\left(\sqrt{R_2^2 - (R_2 \sin\theta)^2} + h - X_A\right)^2 + (k - R_2 \sin\theta - Y_A)^2} \qquad (139)$$

D_2 is D_1 with $X_B Y_B$ substituted for $X_A Y_A$:

$$D_1 = \sqrt{(R_2 \cos\theta + h - X_A)^2 + (k - R_2 \sin\theta - Y_A)^2} \qquad (140)$$

Third quadrant: step θ from π to $\frac{3}{2}\pi$ in $\frac{J}{R_2}$ increments.

Fourth quadrant: step θ from $\frac{3}{2}\pi$ to 2π in $\frac{J}{R_2}$ increments.

Appendix III

PROGRAM LISTINGS FOR CDC6600 COMPUTER

DIFFUSION and APERTURE programs were supplied by General Electric Corporate Research and Development to Wright-Patterson Air Force Base for use on a CDC6600 computer. The DIFFUSION program listing for the CDC-6600 is shown in Figure 58, the APERTURE listing in Figure 59.

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```
      PROGRAM DIFUSR(INPUT,OUTPUT,TAPE1,TAPE2,TAPE3,TAPE4,TAPE5)
C DIFFUSION------A COMPUTOR PROGRAM WHICH CALCULATES THE
C                  DIFFUSION FIELDS AND THE DIFFUSION COUPLED
C                  VOLTAGES INTERIOR TO SEVERAL AIRCRAFT
C                  GEOMETRICAL COMPONENTS.
C
C
C    KEITH J. MAXWELL BLDG 9-209 GENERAL ELECTRIC COMPANY
C    100 WOODLAWN AVE. PITTSFIELD,MASS. 01201
C    PHONE (413)-494-3531.
C
C
C DEVELOPED UNDER CONTRACT F33611-74-C-3060 USAF FLIGHT
C DYNAMICS LABORATORY.
C
C
C THE PROGRAM READS DATA FROM AN EXTERNAL FILE THE NAME
C OF WHICH HAS BEEN SET TO "MAXWELL".THE INPUT DATA SHOULD
C BE ARRANGED AS FOLLOWS FOR FUSELAGE GEOMETRIES'
C
C    LINE NUMBER  100 A
C                 110 A1,R1,R2,X1,Y1,C7,X5,D3,D4,D5,Y5,D6,D7,D8
C                 120 S,L7,D,L3,D3,D4,D5,L4,D6,D7,D8,T8,T9,I4,
C                       G1,G2,G3
C                 130 A
C                 140 ---------SAME AS ABOVE USING 2ND DATA SET------
C LINE NUMBERS MAY BE ADDED INDEFINITELY UNTIL ALL CASES HAVE
C BEEN DESCRIBED .
C
C
C DATA ARRANGEMENT FOR WING,HORIZ STAB,AND VERT STAB
C SHOULD BE AS FOLLOWS'
C
C    LINE NUMBER 100 A
C                110 A2,R,C1,T,S,L7,C7,X5,D3,D4,D5,Y5,D6,D7,D8
C                120 D,L3,D3,D4,D5,L4,D6,D7,D8,T8,T9,I4,
C                      C1,G2,C3
C                130 A
C                140-------SAME AS ABOVE USING 2ND DATA SET--------
C ADDITIONAL LINES OF DATA MAY BE USED UNTIL ALL CASES ARE
C      DESCRIBED. GEOMETRIES MAY BE MIXED OR SEPARATED AS DESIRED.
C
C
C A DESCRIPTION OF THE VARIABL   FOLLOWS'
C
C  A........THE VALUE OF A RC   S THE PROGRAM TO THE APPROPRIATE
C           GEOMETRICAL CONFIGURATION.
C           A=0-----STOP?
C           A=1-----FUSELAGE
C           A=2-----WING
C           A=3-----HORIZ STAB
C           A=4-----VERT STAB
C THE VALUES A1,A2,A3,A4 ARE USED AS A COMPARISON WITH THE
C VALUE OF A TO INSURE THAT THE INPUT DATA CORRESPONDS TO THE
C GEOMETRY SPECIFIED.
C  THE VALUES OF R1 AND R2 ARE THE RADIUS OF CURVATURE OF
C THE TOP CORNERS AND THE BOTTOM CORNERS OF THE FUSELAGE
C RESPECTIVELY.
C X1 AND Y1 ARE THE HEIGHT AND WIDTH OF THE FUSELAGE.
C C7 IS USED TO DECIDE HOW MANY RELOCATIONS OF A CIRCUIT
C CONDUCTOR ARE TO BE MADE.
C X5 AND Y5 ARE THE INITIAL X-Y COORDINATES OF A CIRCUIT
C CONDUCTOR. THE CIRCUIT BEGINS AT A DEPTH OF L3 INSIDE
C THE FUSELAGE AND EXTENDS TO THE DISTANCE L4.
C A SET OF MODIFIERS IS PROVIDED FOR EACH VALUE DESCRIBING
C THE LOCATION OF THE CIRCUIT. THESE MODIFIERS CHANGE THE
C ORIGINAL POSITION OF THE CIRCUIT BY A STEP SIZE GIVEN
C AS '        X-----STEPPED BY AN AMOUNT D3
C             Y-----STEPPED BY AN AMOUNT D6
C            L3-----STEPPED BY AN AMOUNT D3
C            L4-----STEPPED BY AN AMOUNT D6
C STEPPING BEGINS AT     E=04
C                        E=07
C                        E=04
C                        E=07
C FOR THE VARIAALES X,Y,L3,L4 RESPECTIVELT
C STEPPING OF ANY ONE VARIABLE TERMINATES WHEN
C                        E=05--------X=XMAX
C                        E=08--------Y=YMAX
C                       EE=05--------L3=L3MAX
C                        E=08--------L4=L4MAX
C THE PROGRAM EXECUTES OVER THE RANGE OF A DO LOOP
C FROM E=0 TO E=C7.
C THE VARIABLE S SPECIFIES THE AVERAGE SKIN THICKNESS.
C THE VARIABLE D SPECIFIES THE RESISTIVITY IN OHM-CM FOR THE
C TYPE OF MATERIAL WHICH COMPRISES THE SKIN.
C FOR EACH ITERATION A COMPUTATION IS MADE OF THE
C FLUX DENSITY THE TRANSFER INDUCTANCE, ANDTHE
C TRANSFER RESISTANCE.
C ADDITIONALLY ,FOR A SPECIFIED LIGHTNING WAVESHAPE
C A TABULATION OF OPEN CIRCUIT VOLTAGE VS. TIME IS MADE.
C FOR A TIME PERIOD T8 TO T9 IN STEPS OF T8 (USECS).
C THE WAVESHAPE IS CHARACTERIZED BY A DOUBLE EXPONENTIAL
C EQUATION MODIFIED BY THE DIFFUSION TIME CONSTANT.
C THESE EQUATIONS HAVE
C                     AMPLITUDE=I4
C                   EXPONENTS  C1,G2,C3
C
C
```

Figure 58. DIFFUSION Program Listing (Sheet 1 of 9)

```
      C FOR WING--HORIZ--VERT  DATA , )LINES 33-38) ,  R IS THE
      C LEADING EDGE RADIUS,CI IS THE FWD TO AFT LENGTH OF THE
      C MAIN BOX (STRAIGHT SECTION),T IS THE FWD TO AFT LENGTH
      C OF THE FLAPS )WING),TAPERED TRAILING EDGE (HORIZ STAB)
      C OR RUDDER (VERT STAB)
             DIMENSION XMATH(16,16),XMATV1(6,1),XMAT1(16,1),XMATD(16,6)
             DIMENSION XMATH(16,16),XMATJ)16)
             INTEGER A,A1,A2,C7,D4,D5,D7,D8,I,J,K,N,N1,04,05,07,08
             REAL A0,B0,31,B2,B3,B5,B6,B8,B9,C,C1,D,XMATD,D1,D2,D3,D6
             REAL E2,E7,E8,E9,F0,F1,F2,F3,F4,F5,F6,F7,F3,F9,G1,G2,G3
             REAL H1,H2,H3,XMATI,I1,I2,I3,I4,I5,XJ,K1,K2,K3,K4,L1,L2,L3,
            LL4,XL
             REAL XN,XNATH,N7,N8,XNATN,O,O1,O3,O6,P1,P2,Q0,Q1,Q2,Q3,Q4
             REAL R,R1,R2,R5,S,S7,T,T1,T3,T4,T5,T7,T8,T9,XMATV,R
             REAL X,X1,X2,X5,X7,X8,X9,Y,Y1,Y2,Y3,Y5,Y8,Y9,Z2,Z3
             P1=3.1415927
             P2=6.2831853
             E2=2.71828
             N1=16
      C 210  READ("MAXWELL",90) A
        210  READ 101 ,A
        101  FORMAT)6112)
             IF(A.EQ.0)GOTO5000
             GO TO )240,1040,1060,1080,5000),A
      C 240  READ("MAXWELL",90)A1,R1,R2,X1,Y1,C7,X5,03,04,05,Y5,06,07,08
      C      READ( "MAXWELL",90)S,XL,O,L3,03,04,05,L4,06,07,08,T8,T9,I4,
      C     LG1,G2,G3
        240  READ 101 ,A1,04,D5,07,08
             PRINT 107 ,A1,04,05,07,08
        107  FORMAT(10X,4110)
             READ 101 ,C7,04,05,07,08
             PRINT 107 ,C7,04,05,07,08
             READ 103 ,R1,R2,X1,Y1,X5,03
        103  FORMAT(6E12.8)
             PRINT 109 ,R1,R2,X1,Y1,X5,D3
        109  FORMAT(10X,6E14.5)
             READ 103 ,Y5,06,S
             PRINT 109 ,Y5,06,S
             READ 103 ,XL,O,L3,03,L4,06
             PRINT 109 ,XL,O,L3,03,L4,06
             READ 103 ,T8,T9,I4,G1,G2,G3
             PRINT 109 ,T8,T9,I4,G1,G2,G3
             PRINT 262
        262  FORMAT(1H ,17X,"**DIFFUSION--COUPLING--IN--FUSELAGE**")
             PRINT 272
             PRINT 272
        272  FORMAT(1H/)
             PRINT 272
             IEUP=C7+1
             DO 1820 IEDUM=1,IEUP
             IE=IEDUM-1
             REWIND 1
             REWIND 2
             REWIND 3
             REWIND 4
             REWIND 5
             S7=S
             IF(IE)EQ.C7)GOTO 1830
             X9=X5
             Y9=Y5
             L1=L3
             L2=L4
             IF(04.LE.IE)GOTO420
             GOTO440
        420  X9=X5+(03*(IE-04))
             IF(05.LT.IE)GOTO570
        440  IF(07.LE.IE)GOTO460
             GOTO480
        460  Y9=Y5+(D6*(IE-D7))
             IF(08.LT.IE)GOTO590
        480  IF(04.LE.IE)GOTO500
             GOTO520
        500  L1=L3+(03*(IE-04))
             IF(05.LT.IE)GOTO610
        520  IF(07.LE.IE)GOTO540
             GOTO640
        540  L2=L4+(06*(IE-07))
             IF(08.GT.IE)GOTO640
             GOTO630
        570  X9=X5+(D3*D5)
             GOTO440
        590  Y9=Y5+(06*08)
             GOTO480
        610  L1=L3+(03*05)
             GOTO520
        630  L2=L4+(06*08)
        640  H1=X1-R1
             K1=Y1-R1
             H2=R1
             K2=Y1-R1
             H3=R2
             K3=R2
             H4=X1-R2
             K4=R2
             IF(A.EQ.A1)GOTO740
             GOTO6820
        740  CONTINUE
      C RFW
             REWIND 1
             REWIND 2
```

Figure 58. DIFFUSION Program Listing (Sheet 2 of 9)

```
      DO770I=1,N1
      WRITE(1) I,J
  770 CONTINUE
      C=2*(Y1-(R1+R2))+2*(X1-2*R2)+P1*(R1+R2)
      D=C/N1
      X=0
      IUP=N1+1
      DO850IDUM=1,IUP
      J=IDUM-1
      Y=(Y1-R1)-D*I
      IF(Y.LE.R2)GOTO860
      WRITE(2) X,Y
  850 CONTINUE
  860 IF(X.EQ.X1)GOTO930
      X=X1
      IUP=N1+1
      DO920IDUM=1,IUP
      I=IDUM-1
      Y=R2+D*J
      IF(Y.GE.Y1-R1)GOTO930
      WRITE(2) X,Y
  920 CONTINUE
  930 Y=0
      DO980J=1,N1
      X2=R2+D*J
      IF(X2.GE.X1-R2)GOTO990
      WRITE(2) X2,Y
  980 CONTINUE
  990 IF(Y.EQ.Y1)GOTO1060
      Y=Y1
      IUP=N1+1
      DO1050IDUM=1,IUP
      I=IDUM-1
      X=(X1-R1)-D*I
      IF(X.LE.R1)GOTO1060
      WRITE(2) X,Y
 1050 CONTINUE
 1060 KUP=N1+1
      DO1120KDUM=1,KUP
      K=KDUM-1
      T1=K*D/R2
      X2=H3-R2*COS(T1)
      Y2=K3-R2*SIN(T1)
      IF(X2.GE.R2)GOTO1130
      WRITE(2) X2,Y2
 1120 CONTINUE
 1130 CONTINUE
C REM RESET
      KUP=N1+1
      DO1200KDUM=1,KUP
      K=KDUM-1
      T1=K*D/R2
      X2=H4+R2*SIN(T1)
      Y2=K4-R2*COS(T1)
      IF(Y2.GE.R2)GOTO1210
      WRITE(2) X2,Y2
 1200 CONTINUE
 1210 CONTINUE
C REM RESET
      LUP=N1+1
      DO1280LDUM=1,LUP
      L=LDUM-1
      T1=L*D/R1
      X2=H1+R1*COS(T1)
      Y2=K1+R1*SIN(T1)
      IF(X2.LE.H1)GOTO1290
      WRITE(2) X2,Y2
 1280 CONTINUE
 1290 CONTINUE
C REM RESET
      NUP=N1+1
      DO1360NDUM=1,NUP
      N=NDUM-1
      T1=N*D/R1
      X2=H2-R1*SIN(T1)
      Y2=K2+R1*COS(T1)
      IF(Y2.LE.K2)GOTO1370
      WRITE(2) X2,Y2
 1360 CONTINUE
 1370 CONTINUE
C REM RESET
      IF(X5.LE.R2)GOTO1420
      IF(X9.LE.R1)GOTO1430
      IF(X9.GE.H4)GOTO1540
      IF(X9.GE.H1)GOTO1550
 1420 IF(Y9.LE.R2)GOTO1450
 1430 IF(Y9.GE.K1)GOTO1500
      GOTO1600
 1450 IF(X5.EQ.Y9)GOTO1650
      X8=X9
      Y8=Y9
      X7=(R2-SQRT(R2**2-((R2-Y9)**2)))
      GOTO1760
 1500 X8=X9
      X7=R1-(SQRT((R1)**2-((R1-Y1+Y5)**2)))
      Y8=Y9
      GOTO1760
 1540 IF(Y9.LE.R2)GOTO1600
 1550 IF(Y5.GE.K1)GOTO1640
      X8=X9
```

Figure 58. DIFFUSION Program Listing (Sheet 3 of 9)

```
      X7=X1
      Y8=Y9
      GOTO1760
 1680 X8=X9
      X7=X1+(SQRT(R2**2-(R2-Y9)**2))-R2
      Y8=Y9
      GOTO1760
 1640 X8=X9
      X7=X1+(SQRT((R1)**2-(R1-Y1+Y9)**2))-R1
      Y8=Y9
      GOTO1760
 1600 IF(X9.GE.X1/2)GOTO1730
 1490 X8=X9
      X7=0
      Y8=Y9
      GOTO1760
 1730 X8=X9
      X7=X1
      Y8=Y9
 1760 CONTINUE
C REM
C     ASSIGN 1770 TO SW2900
      ASSIGN 1770 TO 1SW290
      GO TO 2740
 1770 CONTINUE
      IF(IE.GT.0)GOTO1800
C     ASSIGN 1790 TO SW3290
      ASSIGN 1790 TO 1SW329
      GO TO 2910
 1790 CONTINUE
 1800 CONTINUE
C     ASSIGN 1808 TO SW4100
      ASSIGN 1808 TO 1SW410
      GO TO 3380
 1808 CONTINUE
C     ASSIGN 1810 TO SW6810
      ASSIGN 1810 TO 1SW4C1
      GO TO 4190
 1810 CONTINUE
 1820 CONTINUE
 1830 GOTO210
 1840 PRINT 1842
 1842 FORMAT(1H ,20X,"**DIFFUSION--COUPLING--IN--WING**")
      GOTO1890
 1860 PRINT 1862
 1862 FORMAT(1H ,10X,"**DIFFUSION--COUPLING--IN--HORIZONTAL
     &--STABILIZER**")
      GOTO1890
 1880 PRINT 1882
 1882 FORMAT(1H ,11X,"DIFFUSION--COUPLING--IN--VERTICAL
     &--STABILIZER**")
 1890 PRINT 272
      PRINT 272
C     READ( "MAXWELL",90)A2,R,C1,T,S,XL,C7,X5,03,04,09,Y9,06,07,08
C     READ( "MAXWELL",98)O,L3,03,04,05,L4,06,07,08,Y8,T9,I4,G1,G2,
C    &G3
      READ 101 ,A2,04,05,07,08
      PRINT 107 ,A2,04,09,07,08
      READ 101 ,C7,04,05,07,08
      PRINT 107 ,C7,04,05,07,08
      READ 103 ,R,C1,T,DUM,X5,03
      PRINT 109 ,R,C1,T,DUM,X5,03
      READ 103 ,Y5,06,S
      PRINT 109 ,Y9,06,S
      READ 103 ,XL,O,L3,03,L4,06
      PRINT 109 ,XL,O,L3,03,L4,06
      READ 103 ,T8,T9,I4,G1,G2,G3
      PRINT 109 ,T8,T9,I4,G1 G2,G3
      S7=S
      IEUP=C7+1
      DO 2720 IEDUM=1,IEUP
      IE=IEDUM-1
      IF(IE.EQ.C7)GOTO 2730
      X9=X5
      Y9=Y5
      L1=L3
      L2=L4
      IF(04.LE.IE)GOTO2060
      GOTO2080
 2060 X9=X5+(03*(IE-04))
      IF(05.LT.IE)GOTO2210
 2080 IF(07.LE.IE)GOTO2100
      GOTO2120
 2100 Y9=Y5+(06*(IE-07))
      IF(08.LT.IE)GOTO2230
 2120 IF(04.LE.IE)GOTO2140
      GOTO2160
 2140 L1=L3+(03*(IE-04))
      IF(05.LT.IE)GOTO2290
 2160 IF(07.LE.IE)GOTO2180
      GOTO2200
 2180 L2=L4+(06*(IE-07))
      IF(08.GT.IE)GOTO2200
      GOTO2270
 2210 X9=X5+(03*05)
      GOTO2080
 2230 Y9=Y5+(06*08)
      GOTO2120
 2290 L1=L3+(03*05)
      GOTO2160
```

Figure 58. DIFFUSION Program Listing (Sheet 4 of 9)

```
 2270 L2=L4+(D6*D8)
 2280 IF(X.EQ.R)GOTO2300
      GOTO4020
 2300 W=ATAN(R/Y)
      REWIND 1
      C=(P2*R)/2*C1*2*(SQRT(R**2+Y**2))
      D=C/N1
      DO2360N=1,N1
      WRITE(1) Y,S
 2360 CONTINUE
      REWIND 2
      IUP=N1+1
      DO2570IDUM=1,IUP
      I=IDUM-1
      X2=(-T8+(I*D*COS(W)))
      Y2=I*D*SIN(W)
      Y3=-Y2
      IF(X2.GT.T)GOTO2440
      GOTO2530
 2440 Y2=R
      Y3=-Y2
      IF(X2.GT.C1)GOTO2480
      GOTO2530
 2480 IF(X2-C1.GT.R)GOTO2580
      Y2=SQRT((R**2)-(X2-C1)**2)
      Y3=-Y2
      IF(X2.GT.C1+R)GOTO2590
 2530 IF(Y2.EQ.0)GOTO2560
      WRITE(2) X2,Y2
      WRITE(2) X2,Y3
      GOTO2570
 2560 WRITE(2) X2,Y2
 2570 CONTINUE
 2580 WRITE(2) C1,R,0
 2590 IF(X5.GT.0)GOTO2640
      X8=X9
      X7=-(T-(ABS(Y9/(TAN(W)))))
      Y8=Y9
      GOTO2670
 2640 X8=X9
      X7=C1+SQRT(R**2-(Y5)**2)
      Y8=Y9
 2670 CONTINUE
C     ASSIGN 2670 TO SW2980
      ASSIGN 2670 TO ISW200
      GO TO 2740
 2670 CONTINUE
      IF(X2.GT.T)GOTO2700
C     ASSIGN 2690 TO SW 3290
      ASSIGN 2690 TO ISW329
      GO TO 2910
 2690 CONTINUE
 2700 CONTINUE
      ASSIGN 2700 TO ISW410
      GO TO 3300
 2700 CONTINUE
C     ASSIGN 2710 TO SW4010
      ASSIGN 2710 TO ISW401
      GO TO 4190
 2710 CONTINUE
 2720 CONTINUE
 2730 GOTO210
 2740 REWIND 1
      REWIND 2
      REWIND 3
      REWIND 4
      DO2890I=1,N1
      READ(1) W,S
      READ(2) X,Y
      WRITE(3) W,X,Y,S
      D1=SQRT((X-X8)**2+(Y-Y8)**2)
      D2=SQRT((X-X7)**2+(Y-Y8)**2)
      IF(D2.EQ.0)GOTO2870
      GOTO2880
 2870 D2=S/2
 2880 WRITE(4) D1,D2
 2890 CONTINUE
C     GO TO SW2900
      GO TO ISW290,(1770,2670)
 2910 REWIND 3
      REWIND 4
      REWIND 1
      REWIND 2
C     READ(3)            ((XMATO(IROW,ICOL),ICOL=1,4),IROW=1,N1)
      DO 2970 IROW=1,N1
      READ(3)                 (XMATO(IROW,ICOL),ICOL=1,4)
      WRITE(1)                (XMATO(IROW,ICOL),ICOL=1,4)
 2970 CONTINUE
      CALL MATZER(XMATV,N1,1)
      DO3010I=1,N1
      XMATV(I,1)=1
 3010 CONTINUE
      CALL MATZER(XMATN,N1,N1)
      CALL MATZER(XMATI,N1,1)
      CALL MATZER(XMATM,N1,N1)
      P2=6.28318
      E2=2.71828
      X5=30000
 3070 FORMAT((4(1H ,G13.5)/))
 3075 FORMAT((5(1H ,G13.5)/))
```

Figure 58. DIFFUSION Program Listing (Sheet 5 of 9)

```
      DO3160I=1,N1
      DO3150J=1,N1
      IF(I.EQ.J)GOTO3140
      R3=SQRT((XMATO(I,2)-XMATO(J,2))**2+(XMATO(I,3)-XMOTO(J,3))**2)
      XMATN(I,J)=ALOG(R5/R3)
      GOTO3150
 3140 XMATN(I,J)=ALOG(R5*2/XMATO(I,4))
 3150 CONTINUE
 3160 CONTINUE
      CALL MATINV(XMATN,XMATM,N1,N1)
      DO 3174 I9=1,N1
      XMATI(I9,1)=0
 3174 CONTINUE
      DO 3179 I9=1,N1
      DO 3178 J9=1,N1
      XMATI(I9,1)=XMATI(I9,1)+XMATN(I9,J9)
 3178 CONTINUE
 3179 CONTINUE
 3189 FORMAT(G13.9/)
      I1=0
      DO3220I=1,N1
      I1=XMATI(I,1)+I1
 3220 CONTINUE
      DO3200I=1,N1
      READ(4) O1,O2
      XMATJ(I)=XMATI(I,1)/I1
      WRITE(2)          I,O1,O2,XMATJ(I)
      WRITE(5)          XMATJ(I)
 3200 CONTINUE
C     GO TO SW3290
      GO TO ISW329,(1798,2690)
 3300 REWIND 3
      REWIND 2
      REWIND 6
      REWIND 5
      DO3830I=1,N1
      READ(3)           XOUNN,X,Y,S
      IF(IE.EQ.0)GOTO3400
      READ(4) O1,O2
      READ(5) XJ
      GOTO3410
 3400 READ(2) II,O1,O2,XJ
 3410 B8=(L1)/(O1*(SQRT((L1**2)+(O1**2))))
      B9=((XL-L1))/(O1*(SQRT((XL-L1)**2+(O1**2))))
      B1=(((1E-5)*XJ)*(B8+B9))
      X6=(L2-L1)*(ABS(X7-X8))
      IF(X.EQ.X9)GOTO3640
      IF(Y.EQ.Y9)GOTO3720
      IF(X.LT.X9)GOTO3560
      IF(Y.LT.Y9)GOTO3520
      T3=ATAN((Y-Y9)/(X-X9))
      Z2=-(SIN(T3))
      Z3=COS(T3)
      GOTO3780
 3520 T3=ATAN((Y9-Y)/(X-X9))
      Z2=SIN(T3)
      Z3=COS(T3)
      GOTO3780
 3560 IF(Y.LT.Y9)GOTO3610
      T3=ATAN((Y-Y9)/(X9-X))
      Z2=-(SIN(T3))
      Z3=-(COS(T3))
      GOTO3780
 3610 T3=ATAN((Y9-Y)/(X9-X))
      Z2=SIN(T3)
      Z3=-(COS(T3))
      GO TO 3780
 3640 T3=PI/2
      IF(Y.LT.Y9)GOTO3690
      Z2=-(SIN(T3))
      Z3=0
      GOTO3780
 3690 Z2=SIN(T3)
      Z3=0
      GOTO3780
 3720 IF(X.GT.X9)GOTO3760
      Z2=0
      Z3=-1
      GOTO3780
 3760 Z2=0
      Z3=1
 3780 B2=B1*Z2
      B3=B1*Z3
      B5=B5+B2
      B6=B6+B3
      B0=SQRT((B5**2)+(B6**2))
 3830 CONTINUE
      IF(B5.EQ.0)GOTO3970
      T4=ATAN(ABS(B6)/ABS(B5))
      IF(B5.GT.0)GOTO3920
      IF(B6.GT.0)GOTO3900
      T5=180+(T4*57.2958)
      GOTO4010
 3900 T5=180-(T4*57.2958)
      GOTO4010
 3920 IF(B6.GT.0)GOTO3950
      T5=360-(T4*57.2958)
      GOTO4010
 3950 T5=T4*57.2958
      GOTO4010
```

Figure 58. DIFFUSION Program Listing (Sheet 6 of 9)

```
3970 IF(B6.GT.0)GOTO4000
     T5=270
     GOTO4010
4000 T5=90
4010 PRINT 4012
4012 FORMAT(1H ,"MAGNETIC.........................FIELD
    &....................COMPUTATION")
     PRINT 272
     PRINT 4032,X9
4032 FORMAT(1H ,"X-COORDINATE=",G13.6)
     PRINT 4034,Y9
4034 FORMAT(1H ,"Y-COORDINATE=",G13.6)
     PRINT 4037,L1
4037 FORMAT(1H ,"Z1-COORDINATE=",G13.5)
     PRINT 4039,L2
4039 FORMAT(1H ,"Z2-COORDINATE=",G13.5)
     PRINT 4042
4042 FORMAT(1H ,1X,"LOOP AREA        B-X              B-Y
    &         B-TOTAL           ANGLE")
     PRINT 4062
4062 FORMAT(1H ,39X,"(WEBERS/METER**2)    (DEGREES)")
     PRINT 272
     PRINT 4082,A4,B5,B6,B8,T5
4082 FORMAT(1H ,5(1H ,G13.6))
     B0=0
     B1=0
     B2=0
     B3=0
     B5=0
     B6=0
C    GO TO SW4100
     GO TO ISW410,(1800,2700)
4190 REWIND 2
     REWIND 4
     REWIND 5
     XN=0
     DO4490I=1,N1
     IF(IE.EQ.0)GOTO4280
     READ(4) D1,D2
     READ(5) XJ
     GOTO4290
4280 READ(2) I1,D1,D2,XJ
4290 Q4=(1E-9)*XJ
     F1=SQRT(L2**2+D2**2)
     F2=SQRT(L1**2+D2**2)
     F3=(L1-XL)
     F4=(L2-XL)
     F5=(F1-(L2)*(ALOG((F1+L2)/D2)))
     F6=(F2-L1*(ALOG((F2+L1)/D2)))
     F7=(SQRT(F3**2+D2**2)-F3*(ALOG((F3+SQRT(F3**2+D2**2))/D2)))
     F8=(SQRT(F4**2+D2**2)-F4*(ALOG((F4+SQRT(F4**2+D2**2))/D2)))
     F9=SQRT(L2**2+D1**2)
     F0=SQRT(L1**2+D1**2)
     Q0=(F9-L2*ALOG((F9+L2)/D1))
     Q1=(F0-L1*ALOG((F0+L1)/D1))
     Q2=(SQRT(F3**2+D1**2)-F3*(ALOG((F3+SQRT(F3**2+D1**2))/D1)))
     Q3=(SQRT(F4**2+D1**2)-F4*(ALOG((F4+SQRT(F4**2+D1**2))/D1)))
     M7=F5-F6+F7-F8
     M8=Q0-Q1+Q2-Q3
     M7=M7*Q4
     M8=M8*Q4
     XM=(XM+(M7-M8))
4490 CONTINUE
     A8=S7*C
     O1=O*(XL/A8)
     PRINT 272
     PRINT 272
     PRINT 4562
4562 FORMAT(1H ,2X,"TRANSFER+-+-+-+-+-+-+-+-+-+-FUNCTION
    &+-+-+-+-+-+-+-+-+-+-COMPUTATION")
     PRINT 272
     PRINT 4572
4572 FORMAT(1H ,8X,"TRANSFER INDUCTANCE
    &                     TRANSFER RESISTANCE")
     PRINT 4582
4582 FORMAT(1H ,13X,"(HENRIES)                         (OHMS)")
     PRINT 4592,XM,O1
4592 FORMAT(1H ,12X,G13.6,22X,G13.6)
     PRINT 4602
4602 FORMAT(1H ,"OPEN         CIRCUIT        VOLTAGE")
     PRINT 272
     PRINT 4614
4614 FORMAT(1H ,"TIME                          VOLTS")
     T7=0
     DO 4720 IDUMMY=1,999
     T7=T7+T8
     IF (T7.GT.T9) GO TO 4721
     I2=I4*((-G1*EXP(-G1*T7))+G2*EXP(-G2*T7))
     I2=I2+I4*((G1+G3)*EXP((-G1-G3)*T7))
     I3=I4*(G2+G3)*EXP((-G2-G3)*T7)
     I5=I2-I3
     E7=O1*I4*(EXP(-G1*T7)-EXP(-G2*T7))*(1-EXP(-G3*T7))
     E8=XM*I5
     E9=E7-E8
     PRINT 4712,T7,E9
4712 FORMAT(1H ,G13.6,3H    ,G13.6)
4720 CONTINUE
4721 CONTINUE
     PRINT 272
     PRINT 4742
```

Figure 58. DIFFUSION Program Listing Sheet 7 of 9)

```
 4742 FORMAT(1H ,75(1H=))                                              00007910
      PRINT 272                                                        00007920
      PRINT 272                                                        00007930
      PRINT 272                                                        00007940
      XN=0                                                             00007950
      01=0                                                             00007960
C     GO TO SW4810                                                     00007970
      GO TO ISW481,(1010,2710)
 4820 PRINT 272                                                        00007980
      PRINT 4832                                                       00007990
 4832 FORMAT(1H ,"DATA READ STATEMENT DOES NOT CONTAIN")               00008000
      PRINT 4842                                                       00008010
 4842 FORMAT(1H ,"VALUES WHICH CORRESPOND TO THIS")                    00008020
      PRINT 4852                                                       00008030
 4852 FORMAT(1H ,"GEOMETRY.CHECK ALL DATA STATEMENTS")                 00008040
      PRINT 4862                                                       00008050
 4862 FORMAT(1H ,"TO BE SURE THAT THEY ARE CONSISTENT")                00008060
      PRINT 4872                                                       00008070
 4872 FORMAT(1H ,"WITH THE GEOMETRY YOU ARE EVALUATING.")              00008080
 5000 STOP                                                             00008090
      END                                                              00008100


      SUBROUTINE MATRIX(IOP,A,B,C,I,J,K,L,M)                           00000300
      REAL A,B,C,TEMP                                                  00000310
      DIMENSION A(I,J),B(I,J),C(I,J)                                   00000320
      DIMENSION LABEL(16)                                              00000330
      GO TO (101,102,103,104,200,300,400), IOP                         00000340
  101 ASSIGN 111 TO IP                                                 00000350
      GO TO 100                                                        00000360
  102 ASSIGN 112 TO IP                                                 00000370
      GO TO 100                                                        00000380
  103 ASSIGN 113 TO IP                                                 00000390
      GO TO 100                                                        00000400
  104 ASSIGN 114 TO IP                                                 00000410
  100 DO 120 I1=1,K                                                    00000420
      DO 120 I2=1,L                                                    00000430
      GO TO IP,(111,112,113,114)                                       00000440
  111 C(I1,I2)=A(I1,I2)+B(I1,I2)                                       00000450
      GO TO 120                                                        00000460
  112 C(I1,I2)=A(I1,I2)-B(I1,I2)                                       00000470
      GO TO 120                                                        00000480
  113 C(I1,I2)=A(I1,I2)*B(I1,I2)                                       00000490
      GO TO 120                                                        00000500
  114 C(I1,I2)=A(I1,I2)/B(I1,I2)                                       00000510
  120 CONTINUE                                                         00000520
      GO TO 500                                                        00000530
  200 DO 210 I1=1,K                                                    00000540
      DO 210 I2=1,L                                                    00000550
      TEMP=0.                                                          00000560
      DO 205 I3=1,M                                                    00000570
  205 TEMP=TEMP+A(I1,I3)*B(I3,I2)                                      00000580
  210 C(I1,I2)=TEMP                                                    00000590
      GO TO 500                                                        00000600
  300 NR=K                                                             00000610
      NC=K                                                             00000620
      DO 21 J1=1,NR                                                    00000630
   21 LABEL(J1)=J1                                                     00000640
      DO 291 J1=1,NR                                                   00000650
      TMP1=0.                                                          00000660
      DO 121 J2=J1,NR                                                  00000670
C     TMP2=CABS(A(J2,J1))                                              00000680
      TMP2=ABS(A(J2,J1))
      IF(TMP2-TMP1) 121,121,1210                                       00000690
 1210 TMP1=TMP2                                                        00000700
      IBIG=J2                                                          00000710
  121 CONTINUE                                                         00000720
      IF(IBIG.EQ.J1) GO TO 201                                         00000730
      DO 141 J2=1,NC                                                   00000740
      TEMP=A(J1,J2)                                                    00000750
      A(J1,J2)=A(IBIG,J2)                                              00000760
  141 A(IBIG,J2)=TEMP                                                  00000770
      I=LABEL(J1)                                                      00000780
      LABEL(J1)=LABEL(IBIG)                                            00000790
      LABEL(IBIG)=I                                                    00000800
  201 TEMP=A(J1,J1)                                                    00000810
      A(J1,J1)=1.0                                                     00000820
      DO 221 J2=1,NC                                                   00000830
  221 A(J1,J2)=A(J1,J2)/TEMP                                           00000840
      DO 281 J2=1,NR                                                   00000850
      IF(J2.EQ.J1) GO TO 281                                           00000860
      TEMP=A(J2,J1)                                                    00000870
      A(J2,J1)=0.                                                      00000880
      DO 241 J3=1,NC                                                   00000890
  241 A(J2,J3)=A(J2,J3)-TEMP*A(J1,J3)                                  00000900
  281 CONTINUE                                                         00000910
  291 CONTINUE                                                         00000920
  301 N1=NR-1                                                          00000930
      DO 391 J1=1,N1                                                   00000940
      DO 321 J2=J1,NR                                                  00000950
      IF(LABEL(J2).NE.J1) GO TO 321                                    00000960
      IF(J2.EQ.J1) GO TO 391                                           00000970
      GO TO 341                                                        00000980
  321 CONTINUE                                                         00000990
  341 DO 361 J3=1,NR                                                   00009000
      TEMP=A(J3,J1)                                                    00009010
      A(J3,J1)=A(J3,J2)                                                00009020
```

Figure 58. DIFFUSION Program Listing (Sheet 8 of 9)

```
  861 A(J3,J2(=TEMP                                          00009030
      LABEL(J2)=LABEL(J1(                                    00009040
  391 CONTINUE                                               00009050
      GO TO 500                                              00009060
  400 DO 410 I1=1,K                                          00009070
      DO 410 I2=1,L                                          00009080
  410 C(I2,I1)=A(I1,I2)                                      00009090
  500 RETURN                                                 00009100
      END                                                    00009110
```

```
      SUBROUTINE MATINV(A,B,IROW,ICOL(                       00000200
      DIMENSION B(IROW,ICOL),B(IROW,ICOL),C(1,1)             00000210
      DO 3540 I9=1 ,IROW                                     00000220
      DO 3530 J9=I , ICOL                                    00000230
      B(I9,J9)=A(I9,J9)                                      00000240
 3530 CONTINUE                                               00000250
 3540 CONTINUE                                               00000260
      CALL MATRIX(6,B,A,C,IROW,ICOL,IROW,ICOL,ICOL)          00000270
      RETURN                                                 00000280
      END                                                    00000290
```

```
      SUBROUTINE MATZER(XM,IROW,ICOL)                        00000110
      DIMENSION XM(IROW,ICOL)                                00000120
      DO 3200 I9=1,IROW                                      00000130
      DO 3190 J9=I,ICOL                                      00000140
      XM(I9,J9)=0                                            00000150
 3190 CONTINUE                                               00000160
 3200 CONTINUE                                               00000170
      RETURN                                                 00000180
      END                                                    00000190
```

Figure 58. DIFFUSION Program Listing (Sheet 9 of 9)

```
      PROGRAM APTUR(INPUT,OUTPUT)
C APERTURE-----A PROGRAM THAT CALCULATES THE MAGNETIC FIELD THAT
C PASSES THROUGH AN APERTURE.  FA FISHER  BLDG 9-209
C GENERAL ELECTRIC COMPANY 100 WOODLAWN AVE PITTSFIELD,MASS 01201
C PHONE (413)-494-4300
C DEVELOPED UNDER CONTRACT F33615-74-C-3000  USAF FLIGHT DYNAMICS LAB
C THE PROGRAM READS DATA FROM AN EXTERNAL FILE, THE NAME OF WHICH
C WILL BE REQUESTED DURING EXECUTION. THE INPUT DATA FILE SHOULD
C BE CONSTRUCTED AS FOLLOWS'
C
C   LINE NUMBER 10 XA,YA,ZA
C               20 L1,L2,ANAH
C               30 HEXT,ANGH
C               40 D1,D2
C               50 D3
C               60 ZPA,ZPB,ZPC
C               70 YPA,YPB,YPC
C               80 XPA,XPB,XPC
C               90 D4
C              100 D5
C              110 PX1,PY1,PZ1,PX2,PY2,PZ2
C              120 PX3,PY3,PZ3,PX4,PY4,PZ4
C
C (LINE NUMBERS NEED NOT BE IDENTICAL TO THOSE ABOVE)
C
C XA,YA,ZA ARE THE COORDINATES IN METERS OF THE CENTER OF THE
C  APERTURE. IT IS LOCATED IN A PLANE PARALLAL TO THE XY PLANE
C
C L1 AND L2 ARE THE LENGTHS IN METERS OF THE AXES OF THE ELLIPTICAL
C APERTURE.  L1=MAJOR AXIS AND = L2=MINOR AXIS.
C  ANAH IS THE ANGLE THAT THE MAJOR AXIS  F THE APERTURE MAKES WITH
C  THE X AXIS.  0 DEGREES IS PARALLEL THE THE POSITIVE X AXIS.
C
C HEXT IS THE STRENGTH IN AMPERES PER METER OF THE EXTERNAL FIELD
C
C ANGH WITH RESPECT TO THE X AXIS, 0 DEGREES=PARALLEL TO X-AXIS.
C D1=1=YES-THERE IS A REFLECTING SURFACE PARALLEL TO THE APERTURE.
C D1=0=NO REFLECTING SURFACE.
C
C D2=Z COORDINATE OF THE REFLECTING SURFACE.  ENTER DUMMY VALUE IF
C D1=0.
C
C D3=1=YES-CALCULATE THE FIELDS OVER A PRESCRIBED VOLUME INSIDE.
C D3=0=NO-SKIP THIS CALCULATION.
C
C ZPA=Z COORDINATE AT WHICH CALCULATION SHOULD START
C ZPB=Z COORDINATE AT WHICH CALCULATION SHOULD END
C ZPC=Z INCREMENT SIZE
C YPA,YPB,YPC,XPA,XPB,XPC ARE SIMILAR FOR X AND Y COORDINATES
C ENTER DUMMY VALUES IF D3=0
C
C D4=1=TABULATE FIELD IN SPHERICAL COORDINATES.
C D4=0=TABULATE IN RECTANGULAR COORDINATES.
C
C D5=1=YES-CALCULATE THE FLUX LINKING A LOOP
C D5=0=NO-SKIP THIS CALCULATION.
C
C PX1,PY1,----PY4,PZ4 ARE THE COORDINATES OF FOUR POINTS THAT
C DEFINE THE LOOP.  THEY MUST GO AROUND THE LOOP IN CONSUCUTIVE
C ORDER.  ADDITIONAL LOOPS MAY BE DEFINED BY ADDITIONAL DATA IN
C THE SAME FORMAT.  DUMMY VALUES ARE NOT REQUIRED IF D5=0
C ''''''''''''''''''''''''''''''''''''''''''''''
      DIMENSION MM(13,13)
      DIMENSION TBGA(13)
      DIMENSION PATMAT(13)
      REAL L1,L2,MU1,MU2,MU3,MU,MUX,MUY,MU2
   30 PRINT 115
C CARRIAGE CONTROL FORMAT STATEMENTS
  110 FORMAT(1H*)
  115 FORMAT(1H0)
  120 FORMAT(1H )
  122 FORMAT(1H&)
  123 FORMAT(1H+)
C OUTPUT DATA FORMATS
  130 FORMAT(6E12.5)
C DATA HEADING FORMATS
  140 FORMAT(" APERTURE COORDINATES--X=",1E12.3,"  METERS")
  145 FORMAT("                          Y=",1E12.3,"  METERS")
  150 FORMAT("                          Z=",1E12.3,"  METERS")
  155 FORMAT(" APERTURE DIMENSIONS--MAJOR AXIS=",1E12.3,
     &"  METERS")
  160 FORMAT("                      MINOR AXIS=",1E12.3,
     &"  METERS")
  165 FORMAT(" APIRTURE INCLINED",1E12.5,"  DEGREES FROM X AXIS")
  170 FORMAT(" EXTERNAL MAGNETIC FIELD=",1E12.3,
     &"  AMPERES PER METER")
  175 FORMAT(" AND INCLINED",1E12.5,"  DEGREES FROM THE X AXIS")
  180 FORMAT(" THERE IS NO REFLECTING SURFACE")
  185 FORMAT(" THERE IS A REFLECTING SURFACE LOCATED AT Z=",
     &1E12.3, "   METERS")
  188 FORMAT(" LOOP NUMBER  ",I3)
  190 FORMAT(" LOOP ARIA=",1E12.5,"   SQUARE METERS")
  192 FORMAT(" TOTAL FLUX=",1E12.5,"  WEBERS")
  195 FORMAT(" OUT OF DATA")
  215 FORMAT(" POINT      X            Y            Z")
  220 FORMAT(I5,3E12.3)
C     READ(INFILE,230,END=1900)LINE,XA,YA,ZA
C     READ (INFILE,240,END=1960)LINE,L1,L2,ANGA
      READ 103 ,XA,YA,ZA
```

Figure 59. Computer Program APERTURE (Sheet 1 of 7)

```
      103 FORMAT(6E12.0)
          READ 103 ,L1,L2,ANGA
          PRINT 140,XA
          PRINT 149,YA
          PRINT 150,ZA
          PRINT 155,L1
          PRINT 160,L2
          PRINT 165,ANGA
C         READ(INFILE,200,END=1960)LINE,NEXT,ANGN
          READ 103 ,NEXT,ANGN
          PRINT 115
          PRINT 170,NEXT
          PRINT 175,ANGN
          PRINT 115
C         READ(INFILE,200,END=1900)LINE,D1,D2
          READ 103 ,D1,D2
      275 IF(D1)280,280,290
      280 PRINT 180
          PRINT 115
      285 GOTO295
      290 PRINT 185,D2
          PRINT 115
      295 CONTINUE
C         READ(INFILE,200,END=1960)LINE,D3
C         READ(INFILE,200,END=1900)LINE,ZPA,ZPB,ZPC
C         READ(INFILE,200,END=1900)LINE,YPP,YPB,YPC
C         READ(INFILE,200,END=1900)LINE,XPA,XPB,XPC
C         READ(INFILE,200,END=1960)LINE,D4
          READ 103 ,D3
          READ 103 ,ZPA,ZPB,ZPC
          READ 103 ,YPA,YPB,YPC
          READ 103 ,XPA,XPB,XPC
          READ 103 ,D4
          PI=3.14159265
          CALL SHAPE(L1,L2,A11,A22)
          IF(D3)1952,1952,2200
     2200 IF(D4)2201,2201,2206
     2201 PRINT 2202
     2202 FORMAT("    X          Y    S         Z                N-X
         &    @   N-Y              N-Z")
          GOTO 2204
     2206 PRINT 2207
     2207 FORMAT("    X          Y              Z                NTOT
         &            LAT              LONG")
     2204 PRINT 120
          GOTO 2450
     2205 PRINT 120
     2450 CONTINUE
          J1=IFIX((ZPB-ZPA)/ZPC)+1
          J2=IFIX((YPB-YPA)/YPC)+1
          J3=IFIX((XPB-XPA)/XPC)+1
          DO1950 I3=1,J3,1
          DO1950 I2=1,J2,1
          DO1950 I1=1,J1,1
          XP1=XPA+(I3-1)*XPC
          YP1=YPA+(I2-1)*YPC
          ZP1=ZPA+(I1-1)*ZPC
     1002 CONTINUE
     1199 CONTINUE
     1750 CALL MAGFLO(ANGA,ANGN,XP1,YP1,ZP1,XA,YA,ZA,NEXT,A11,A22,
         &HPX1,HPY1,HPZ1,L1,L2,D1,D2)
     3370 IF(D4)1210,1210,4000
     1210 PRINT 1220,XP1,YP1,ZP1,HPX1,HPY1,HPZ1
          GOTO 1950
     1220 FORMAT(6E12.3)
     4000 D=SORT(HPX1*HPX1+HPZ1*HPZ1)
     4002 IF(ABS(HPY1)-ABS(D)) 4004,4004,4010
     4004 ANG1=90-57.2957795*(ATAN(HPY1/D))
     4006 GOTO 4012
     4010 ANG1=57.2957795*(ATAN(D/HPY1))
     4012 IF(ABS(HPX1)-ABS(HPZ1)) 4014,4014,4020
     4014 ANG2=90-57.2957795*(ATAN(HPX1/HPZ1))
     4016 GOTO 4030
     4020 ANG2=57.2957795*(ATAN(HPZ1/HPX1))
     4030 IF(HPY1) 4050,4050,4040
     4040 GOTO 4110
     4050 ANG1=180-ANG1
     4110 CONTINUE
     4120 IF(HPX1)4180,4130,4130
     4130 IF(HPZ1)4160,4140,4140
     4140 ANG2=ANG2
     4150 GOTO 4215
     4160 ANG2=-ANG2
     4170 GOTO 4215
     4180 IF(HPZ1)4210,4190,4190
     4190 ANG2=180-ANG2
     4200 GOTO 4215
     4210 ANG2=-(180-ANG2)
     4215 CONTINUE
          HPT=SORT(HPX1*HPX1+HPY1*HPY1+HPZ1*HPZ1)
     4230 PRINT 1220,XP1,YP1,ZP1,HPT,ANG1,ANG2
     1950 CONTINUE
     1952 PRINT 110
C         READ(INFILE,200,END=1960)LINE,D5
          READ 103 ,D5
     2100 IF(D5)1400,1400,1955
     1955 CONTINUE
          JK=0
     1957 CONTINUE
C         READ(INFILE,200,END=1900)LINE,PX1,PY1,PZ1,PX2,PY2,PZ2
```

Figure 59. Computer Program APERTURE (Sheet 2 of 7)

```
C     READ(INFILE,200,END=1960)LINE,PX3,PY3,PZ3,PX4,PY4,PZ4
      READ 103 ,PX1,PY1,PZ1,PX2,PY2,PZ2
      IF (PX1+100000.0) 1400,1406,1950
 1950 READ 133 ,PX3,PY3,PZ3,PX4,PY4,PZ4
C THESE ARE THE SIDES OF THE QUADRILATERAL
 2110 CONTINUE
      X21=PX2-PX1
      X32=PX3-PX2
      X43=PX4-PX3
      X14=PX1-PX4
      Y21=PY2-PY1
      Y32=PY3-PY2
      Y43=PY4-PY3
      Y14=PY1-PY4
      Z21=PZ2-PZ1
      Z32=PZ3-PZ2
      Z43=PZ4-PZ3
      Z14=PZ1-PZ4
C THIS IS A DIAGONAL OF THE QUADRILATERAL
      X31=PX3-PX1
      Y31=PY3-PY1
      Z31=PZ3-PZ1
      T21=SQRT(X21*X21+Y21*Y21+Z21*Z21)
      T32=SQRT(X32*X32+Y32*Y32+Z32*Z32)
      T43=SQRT(X43*X43+Y43*Y43+Z43*Z43)
      T14=SQRT(X14*X14+Y14*Y14+Z14*Z14)
      T31=SQRT(X31*X31+Y31*Y31+Z31*Z31)
      S1=(T21+T32+T31)/2
      A1=SQRT(S1*(S1-T21)*(S1-T32)*(S1-T31))
      S2=(T43+T14+T31)/2
      A2=SQRT(S2*(S2-T43)*(S2-T14)*(S2-T31))
      AREA=A1+A2
C THESE ARE THE MIDPOINTS OF THE ENDS OF THE QUADRILATERAL
      XPN1=PX1+X21/2
      YPN1=PY1+Y21/2
      ZPN1=PZ1+Z21/2
      XPN2=PX4-X43/2
      YPN2=PY4-Y43/2
      ZPN2=PZ4-Z43/2
      XPN21=XPN2-XPN1
      YPN21=YPN2-YPN1
      ZPN21=ZPN2-ZPN1
      TPM=SQRT(XPN21*XPN21+YPN21*YPN21+ZPN21*ZPN21)
C THESE ARE THE COMPONENTS OF THE NORMAL VECTOR
      NU1=Y21*Z32-Y32*Z21
      NU2=-X21*Z32+X32*Z21
      NU3=X21*Y32-X32*Y21
      TEMP=NU1*NU1+NU2*NU2+NU3*NU3
      NU=SQRT(TEMP)
C THESE ARE THE COMPONO    . OF THE UNIT NORMAL VECTOR
      NUX=NU1/NU
      NUY=NU2/NU
      NUZ=NU3/NU
 3550 CONTINUE
 3560 DO 3000 N2=1,13,1
 3570 DO3000 N1=1,13,1
      XP5=PX1+X21*(N1-1)/12
      YP5=PY1+Y21*(N1-1)/12
      ZP5=PZ1+Z21*(N1-1)/12
      XP6=PX2+X32*(N2-1)/12
      YP6=PY2+Y32*(N2-1)/12
      ZP6=PZ2+Z32*(N2-1)/12
      XP7=PX4-X43*(N1-1)/12
      YP7=PY4-Y43*(N1-1)/12
      ZP7=PZ4-Z43*(N1-1)/12
      XP8=PX1-X14*(N2-1)/12
      YP8=PY1-Y14*(N2-1)/12
      ZP8=PZ1-Z14*(N2-1)/12
      X75=XP7-XP5
      Y75=YP7-YP5
      Z75=ZP7-ZP5
      X86=XP8-XP6
      Y86=YP8-YP6
      Z86=ZP8-ZP6
      T86=SQRT(X86*X86+Y86*Y86+Z86*Z86)
      XPL=XP8-X86*(N1-1)/12
      YPL=YP8-Y86*(N1-1)/12
      ZPL=ZP8-Z86*(N1-1)/12
      CALL MAGFLD(ANGA,ANGM,XPL,YPL,ZPL,XA,YA,ZA,HEXT,A11,A22,HPX1
     & ,HPY1,HPZ1,L1,L2,D1,D2)
      HNP=HPX1*NUX+HPY1*NUY+HPZ1*NUZ
C THESE ARE THE HN'S AT THE VARIOUS POINTS OF THE QUADRILATERAL
      HN(N1,N2)=HNP
C THESE ARE THE DISTANCES TOP TO BOTTOM ALONG THE QUADRILATERAL
      T86A(N2)=T86
 3000 CONTINUE
 3850 DO3950 N2=1,13,1
      DELTA1=T86A(N2)/12
C THIS EVALUATES FLUX ALONG THE LINES IN THE DIRECTION P1---P2
C AND P4---P3
      PATH=HN(1,N2)+HN(2,N2)*5+HN(3,N2)+HN(4,N2)*6+HN(5,N2)+HN(6,
     &N2)*5+ HN(7,N2)*2+HN(8,N2)*5+HN(9,N2)+HN(10,N2)*6+
     & HN(11,N2)+HN(12,N2)*5+HN(13,N2)
      PATHA(N2)=0.3*DELTA1*PATH
 3950 CONTINUE
      DELTA2=TPM/12
C THIS EVALUATES THE RESULTANT FLUX IN THE DIRECTION P1---P4
C AND P2---P3
      HTOT=PATHA(1)+PATHA(2)*5+PATHA(3)+PATHA(4)*6+PATHA(5)+
     & PATHA(6)*5+PATHA(7)*2+PATHA(8)*5+PATHA(9)+PATHA(10)*6+
```

Figure 59. Computer Program APERTURE (Sheet 3 of 7)

```
     1 PATMA(11)+PATMA(12)*5+PATMA(13)                                 00003880
      NTOT=0.5*DELTA2*NTOT                                              00003890
      BIOT=NTOT*4*PI*1E-7                                               00003900
      JX=JX+1                                                           00003910
 4315 PRINT 100,JX                                                      00003920
      PRINT 120                                                         00003930
      PRINT 210                                                         00003940
      PRINT 120                                                         00003950
      J=1                                                               00003960
      PRINT 220,J,PX1,PY1,PZ1                                           00003970
      J=2                                                               00003980
      PRINT 220,J,PX2,PY2,PZ2                                           00003990
      J=3                                                               00004000
      PRINT 220,J,PX3,PY3,PZ3                                           00004010
      J=4                                                               00004020
      PRINT 220,J,PX4,PY4,PZ4                                           00004030
      PRINT 120                                                         00004040
 4320 PRINT 190,AREA                                                    00004050
 4330 PRINT 192,BIOT                                                    00004060
 4340 PRINT 110                                                         00004070
      GOTO 1957                                                         00004080
 4309 PRINT 110                                                         00004090
 1960 PRINT 195                                                         00004100
 1430 STOP                                                              00004110
      END                                                               00004120


      SUBROUTINE SHAPE(L1,L2,A11,A22)                                   00004130
      REAL L1,L2                                                        00004140
      PI=3.14159265                                                     00004150
      E1=1-(L2/L1)**2                                                   00004160
      E2=SQRT(E1)                                                       00004170
      TEMP=L2/L1
      IF (TEMP.LT.0.0) GO TO 3130                                       00004180
      IF (TEMP.GT.1.0) GO TO 3160                                       00004170
C AAAAAAAAAAAAAAAAAAAAAAAAAAAAA                                         00004200
C CEL1(1,E2) AND CEL1(2,E2) ARE MATH LIBRARY ROUTINES THAT              00004210
C EVALUATE THE ELLIPTIC INTEGRALS OF THE FIRST AND SECOND KINDS.        00004220
      CALL CEL1(Y1,E2,IER)
      E3=1.0-E2*E2
      CALL CEL2(Y2,E2,1.0,E3,IER)
C AAAAAAAAAAAAAAAAAAAAAAAAAAAAA                                         00004250
      CON1=2*PI*(L1/2)**3/3                                             00004260
      A11=CON1*E1/(Y1-Y2)                                               00004270
      A22=CON1*E1*(1-E1)/(Y2-(1-E1)*Y1)                                 00004280
      A33=CON1*(1-E1)/Y2                                                00004290
      RETURN                                                            00004300
 3130 PRINT 3140                                                        00004310
 3140 FORMAT(" L2/L1 IS NEGATIVE. THIS IS AN ERROR")                    00004320
      RETURN                                                            00004330
 3160 PRINT 3170                                                        00004340
 3170 FORMAT(" L2 IS LARGER THAN L1.  THIS IS AN ERROR")                00004350
      STOP                                                              00004360
      END                                                               00004370


      SUBROUTINE MAGFLD(ANGA,ANGH,XP1,YP1,ZP1,XA,YA,ZA,HEXT,A11,        00004380
     1 A22,HPX1,HPY1,HPZ1,L1,L2,D1,D2)                                  00004390
      REAL K1,K2,K5,K6,L1,L2                                            00004400
      N=0                                                               00004410
      RAD=57.2957795                                                    00004420
      PI=3.14159265                                                     00004430
      K1=COS(ANGA/RAD)                                                  00004440
      K2=SIN(ANGA/RAD)                                                  00004450
C CALCULATION OF SHIFTED COORDINATES OF POINT UNDER INVESTIGATION       00004460
      XP2=XP1*K1+YP1*K2                                                 00004470
      YP2=-XP1*K2+YP1*K1                                                00004480
      ZP2=ZP1                                                           00004490
      HPX2=0                                                            00004500
      HPY2=0                                                            00004510
      HPZ2=0                                                            00004520
      ZAA=ZA                                                            00004530
C CALCULATION OF DISTANCES FROM APERTURE TO POINT UNDER STUDY           00004540
 9130 XC=XP2-XA                                                         00004550
 9140 YC=YP2-YA                                                         00004560
 9150 ZC=ZP2-ZAA                                                        00004570
 9160 C1=XC*XC+YC*YC+ZC*ZC+L1*L1/4                                      00004580
      C2=XC*XC+YC*YC+ZC*ZC+L2*L2/4                                      00004590
C CALCULATION OF H FIELD PARALLEL TO AXES OF APERTURE                   00004600
      CON1=1.45035                                                      00004610
      CON2=1.834558                                                     00004620
      CON3=2.034727                                                     00004630
C CALCULATION OF FIELD PARALLEL TO AXES OF APERTURE                     00004640
      AMAH=(ANGH-ANGA)/RAD                                              00004650
      HMAJ=HEXT*COS(AMAH)                                               00004660
      HMIN=HEXT*SIN(AMAH)                                               00004670
C CALCULATION OF ROTATED COMPONENTS OF MAGNETIC FIELD                   00004680
      K5=-A11*HMAJ/(4*PI*L1)                                            00004690
      K6=-A22*HMIN/(4*PI*L2)                                            00004700
      C3=XC*L1                                                          00004710
 9165 IF(ABS(C3)-1E-5) 9180,9170,9170                                   00004720
 9170 F3=C3*C3                                                          00004730
      F7=1+2*F3+5*F3*F3+7*F3*F3*F3                                      00004740
      GOTO 9190                                                         00004750
 9180 F7=1                                                              00004760
```

Figure 59. Computer Program APERTURE (Sheet 4 of 7)

```
 9190 C5=C3/C1
 9195 IF(ABS(C5)-1E-5) 9210,9200,9200
 9200 C7=C5*C9
      F1=C5*(1+CON1*C7+CON2*C7*C7+CON3*C7*C7*C7)
      GOTO 9220
 9210 F1=C9
 9220 C4=YC*L2
 9230 IF(ABS(C4)-1E-5) 9290,9240,9240
 9240 F5=C4*C4
      F8=1+2*F5+9*F5*F5+7*F5*F5*F5
      GOTO 9260
 9250 F8=1
 9260 C6=C4/C2
 9270 IF(ABS(C6)-1E-9) 9300,9280,9280
 9280 C8=C6*C6
      F2=C6*(1+CON1*C8+CON2*C8*C8+CON3*C8*C8*C8)
      GOTO 9320
 9300 F2=C6
 9320 CONTINUE
      G1=3*K9*XC/C1**1.9
      G2=3*K6*XC/C2**1.5
      G3=K5*L1/C1**1.9
      G4=3*K5*YC/C1**1.9
      G5=3*K6*YC/C2**1.5
      G6=K8*L2/C2**1.5
      G7=3*K9*ZC/C1**1.5
      G8=3*K6*ZC/C2**1.5
      HPX=G1*F1+G2*F2-G3*F7
      HPY=G4*F1+G5*F2-G6*F8
      HPZ=G7*F1+G8*F2
 9600 IF(D1) 9766,9760,9602
 9602 HPX2=HPX2+HPX
      HPY2=HPY2+HPY
      HPZ2=HPZ2+HPZ
 9690 IF(ABS(HPX)-.005*ABS(HPX2))9700,9700,9720
 9700 IF(ABS(HPY)-.005*ABS(HPY2))9710,9710,9720
 9710 IF(ABS(HPZ)-.005*ABS(HPZ2))9770,9770,9726
 9720 IF(N-10)9730,9730,9770
 9730 N=N+1
 9740 ZAA=ZAA-((-1)**N)*2*N*D2
      GOTO9130
 9760 HPX2=HPX2+HPX
      HPY2=HPY2+HPY
      HPZ2=HPZ2+HPZ
 9770 CONTINUE
C CALCULATION OF COMPONENTS OF MAGNETIC FIELD ROTATED BACK
C TO THE REFERENCE AXES
      HPX1=HPX2*K1-HPY2*K2
      HPY1=HPX2*K2+HPY2*K1
      HPZ1=HPZ2
      RETURN
 5230 STOP
      END
C
C     .....................................................................
C
C        SUBROUTINE CEL1
C
C        PURPOSE
C           CALCULATE COMPLETE ELLIPTIC INTEGRAL OF FIRST KIND
C
C        USAGE
C           CALL CEL1(RES,AK,IER)
C
C        DESCRIPTION OF PARAMETERS
C           RES    - RESULT VALUE
C           AK     - MODULUS (INPUT)
C           IER    - RESULTANT ERROR CODE WHERE
C                    IER=0  NO ERROR
C                    IER=1  AK NOT IN RANGE -1 TO +1
C
C        REMARKS
C           FOR AK=+1,-1 THE RESULT IS SET TO 1.E75.
C           FOR MODULUS AK AND COMPLEMENTARY MODULUS CK,
C           EQUATION AK*AK+CK*CK=1.0 IS USED.
C           AK MUST BE IN THE RANGE -1 TO +1
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        METHOD
C           DEFINITION
C           CEL1(AK)=INTEGRAL(1/SQRT((1+T*T)*(1+(CK*T)**2)), SUMMED
C           OVER T FROM 0 TO INFINITY).
C           EQUIVALENT ARE THE DEFINITIONS
C           CEL1(AK)=INTEGRAL(1/(COS(T)SQRT(1+(CK*TAN(T))**2)),SUMMED
C           OVER T FROM 0 TO PI/2),
C           CEL1(AK)=INTEGRAL(1/SQRT(1-(AK*SIN(T))**2),SUMMED OVER T
C           FROM 0 TO PI/2), WHERE K=SQRT(1.-CK*CK).
C           EVALUATION
C           LANDENS TRANSFORMATION IS USED FOR CALCULATION.
C           REFERENCE
C           R.BULIRSCH, "NUMERICAL CALCULATION OF ELLIPTIC INTEGRALS
C           AND ELLIPTIC FUNCTIONS", HANDBOOK SERIES SPECIAL FUNCTIONS,
C           NUMERISCHE MATHEMATIK VOL. 7, 1965, PP. 78-98.
C
C     .....................................................................
C
```

Figure 59. Computer Program APERTURE (Sheet 5 of 7)

```
      SUBROUTINE CEL1(RES,AK,IER)                                       CEL10460
C                                                                       CEL10470
      IER=0                                                             CEL10480
C                                                                       CEL10490
C        TEST MODULUS                                                   CEL10500
C                                                                       CEL10510
      GEO=1.-AK*AK                                                      CEL10520
      IF(GEO)1,2,3                                                      CEL10530
    1 IER=1                                                             CEL10540
      RETURN                                                            CEL10550
C                                                                       CEL10560
C        SET RESULT VALUE =OFLOW                                        CEL10570
C                                                                       CEL10580
    2 RES=1.E75                                                         CEL10590
      RETURN                                                            CEL10600
    3 GEO=SQRT(GEO)                                                     CEL10610
      ARI=1.                                                            CEL10620
    4 AARI=ARI                                                          CEL10630
      TEST=AARI*1.E-4                                                   CEL10640
      ARI=GEO+ARI                                                       CEL10650
C                                                                       CEL10660
C        TEST OF ACCURACY                                               CEL10670
C                                                                       CEL10680
      IF(AARI-GEO-TEST)6,6,5                                            CEL10690
    5 GEO=SQRT(AARI*GEO)                                                CEL10700
      ARI=0.5*ARI                                                       CEL10710
      GO TO 4                                                           CEL10720
    6 RES=3.14159265/ARI                                                CEL10730
      RETURN                                                            CEL10740
      END                                                               CEL10750
C                                                                       CEL20010
C     ..................................................................CEL20020
C                                                                       CEL20030
C        SUBROUTINE CEL2                                                CEL20040
C                                                                       CEL20050
C        PURPOSE                                                        CEL20060
C           COMPUTES THE GENERALIZED COMPLETE ELLIPTIC INTEGRAL OF      CEL20070
C           SECOND KIND.                                                CEL20080
C                                                                       CEL20090
C        USAGE                                                          CEL20100
C           CALL CEL2(RES,AK,A,B,IER)                                   CEL20110
C                                                                       CEL20120
C        DESCRIPTION OF PARAMETERS                                      CEL20130
C           RES    - RESULT VALUE                                       CEL20140
C           AK     - MODULUS (INPUT)                                    CEL20150
C           A      - CONSTANT TERM IN NUMERATOR                         CEL20160
C           B      - FACTOR OF QUADRATIC TERM IN NUMERATOR              CEL20170
C           IER    - RESULTANT ERROR CODE WHERE                         CEL20180
C                    IER=0  NO ERROR                                    CEL20190
C                    IER=1  AK NOT IN RANGE -1 TO +1                    CEL20200
C                                                                       CEL20210
C        REMARKS                                                        CEL20220
C           FOR AK = +1,-1 THE RESULT VALUE IS SET TO 1.E75 IF B IS     CEL20230
C           POSITIVE, TO -1.E75 IF B IS NEGATIVE.                       CEL20240
C           SPECIAL CASES ARE                                           CEL20250
C           K(K) OBTAINED WITH A = 1, B = 1                             CEL20260
C           E(K) OBTAINED WITH A = 1, B = CK*CK WHERE CK IS             CEL20270
C           COMPLEMENTARY MODULUS.                                      CEL20280
C           B(K) OBTAINED WITH A = 1, B = 0                             CEL20290
C           D(K) OBTAINED WITH A = 0, B = 1                             CEL20300
C           WHERE K, E, B, D DEFINE SPECIAL CASES OF THE GENERALIZED    CEL20310
C           COMPLETE ELLIPTIC INTEGRAL OF SECOND KIND IN THE USUAL      CEL20320
C           NOTATION, AND THE ARGUMENT K OF THESE FUNCTIONS MEANS       CEL20330
C           THE MODULUS.                                                CEL20340
C                                                                       CEL20350
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  CEL20360
C           NONE                                                        CEL20370
C                                                                       CEL20380
C        METHOD                                                         CEL20390
C           DEFINITION                                                  CEL20400
C           RES=INTEGRAL((A+B*T*T)/(SQRT((1+T*T)*(1+(CK*T)**2))*(1+T*T))CEL20410
C           SUMMED OVER T FROM 0 TO INFINITY).                          CEL20420
C           EVALUATION                                                  CEL20430
C           LANDENS TRANSFORMATION IS USED FOR CALCULATION.             CEL20440
C           REFERENCE                                                   CEL20450
C           R.BULIRSCH, "NUMERICAL CALCULATION OF ELLIPTIC INTEGRALS    CEL20460
C           AND ELLIPTIC FUNCTIONS", HANDBOOK SERIES SPECIAL FUNCTIONS, CEL20470
C           NUMERISCHE MATHEMATIK VOL. 7, 1965, PP. 78-90.              CEL20480
C                                                                       CEL20490
C     ..................................................................CEL20500
C                                                                       CEL20510
      SUBROUTINE CEL2(RES,AK,A,B,IER)                                   CEL20520
C                                                                       CEL20530
      IER=0                                                             CEL20540
C                                                                       CEL20550
C        TEST MODULUS                                                   CEL20560
C                                                                       CEL20570
      GEO=1.-AK*AK                                                      CEL20580
      IF(GEO)1,2,8                                                      CEL20590
    1 IER=1                                                             CEL20600
      RETURN                                                            CEL20610
C                                                                       CEL20620
C        SET RESULT VALUE = OVERFLOW                                    CEL20630
C                                                                       CEL20640
    2 IF(B)3,5,4                                                        CEL20650
    3 RES=-1.E75                                                        CEL20660
      RETURN                                                            CEL20670
    4 RES=1.E75                                                         CEL20680
      RETURN                                                            CEL20690
    5 RES=A                                                             CEL20700
```

Figure 59. Computer Program APERTURE (Sheet 6 of 7)

```
      RETURN                                                            CEL20710
C                                                                       CEL20720
C     COMPUTE INTEGRAL                                                  CEL20730
C                                                                       CEL20740
    6 GEO=SQRT(GEO)                                                     CEL20750
      ARI=1.                                                            CEL20760
      AA=A                                                              CEL20770
      AN=A+B                                                            CEL20780
      W=B                                                               CEL20790
    7 W=W+AA*GEO                                                        CEL20800
      W=W+W                                                             CEL20810
      AA=AN                                                             CEL20820
      AARI=ARI                                                          CEL20830
      ARI=GEO+ARI                                                       CEL20840
      AN=W/ARI+AN                                                       CEL20850
C                                                                       CEL20860
C     TEST OF ACCURACY                                                  CEL20870
C                                                                       CEL20880
      IF(AARI-GEO-1.E-6*AARI)9,9,8                                      CEL20890
    8 GEO=SQRT(GEO*AARI)                                                CEL20900
      GEO=GEO+GEO                                                       CEL20910
      GO TO 7                                                           CEL20920
    9 RES=.78539816*AN/ARI                                              CEL20930
      RETURN                                                            CEL20940
      END                                                               CEL20950
```

Figure 59. Computer Program APERTURE (Sheet 7 of 7)